An introduction to ultra-intense lasers

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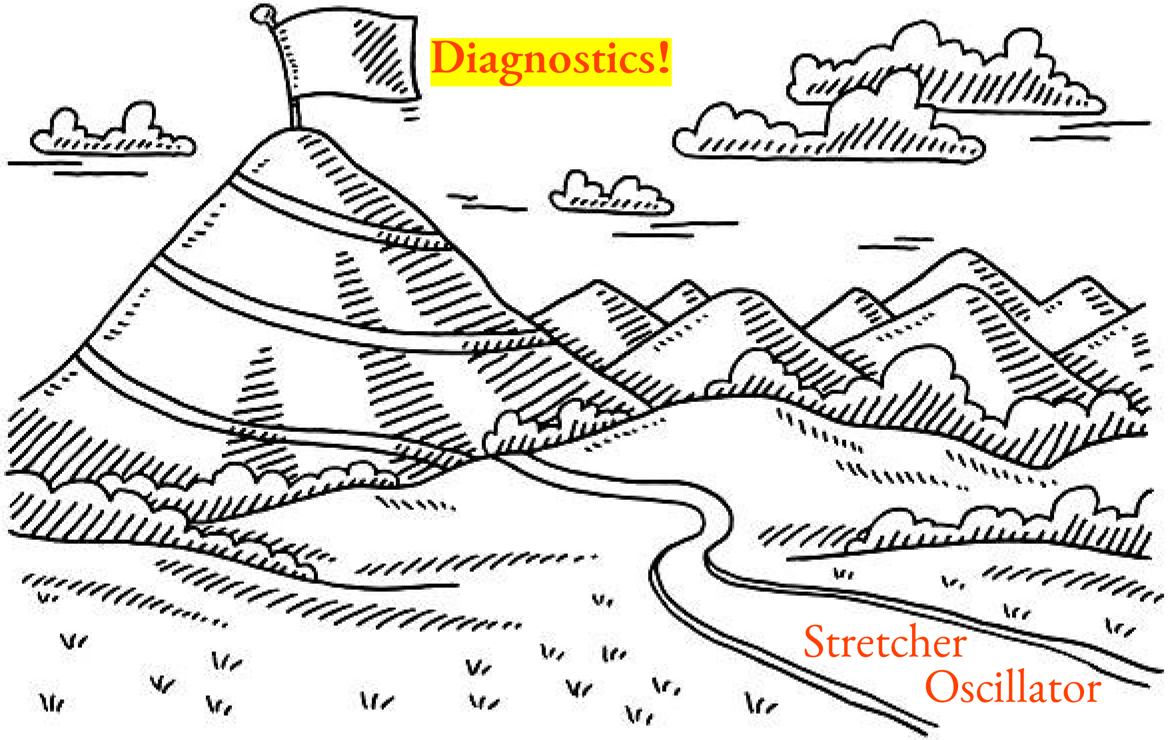
Outline



Ultrashort, high power laser

Focusing and transverse quantities: wavefront (transverse phase), ...

Longitudinal quantities: time duration, spectrum, spectral phase, ...

Motivations (1): Laser-driven particle acceleration at ILIL

Ultrashort/ultraintense laser interaction with matter can be fruitfully employed to accelerate charged particles to high energy

The underlying processes are different for electrons and protons/light ions

Compact, "table-top" accelerators (thanks to the huge accelerator gradients)

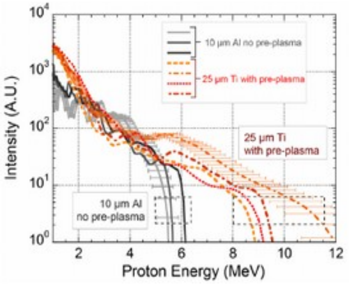
Proton (and light ion) beams can be accelerated up to ~10MeV/nucleon via the so-called **TNSA process** (and others)

OPEN

Enhanced laser-driven proton acceleration via improved fast electron heating in a controlled pre-plasma

Leonida A. Gizzi[1,2], Elisabetta Boella[3,4], Luca Labate[1,

Scientific Reports | (2021) 11:1372



PHYSICAL REVIEW APPLIED **21**, 064020 (2024)

Quantitative elemental analysis of a specimen in air via external beam laser-driven particle-induced x-ray emission with a compact proton source

Martina Salvadori,[1,*] Fernando Brandi,[1,†] Luca Labate,[1,‡,§] Federica Baffigi,[1] Lorenzo Fulgentini,[1] Pietro Galizia,[2] Petra Koester,[1] Daniele Palla,[1] Diletta Sciti,[2] and Leonida A. Gizzi[1,§]

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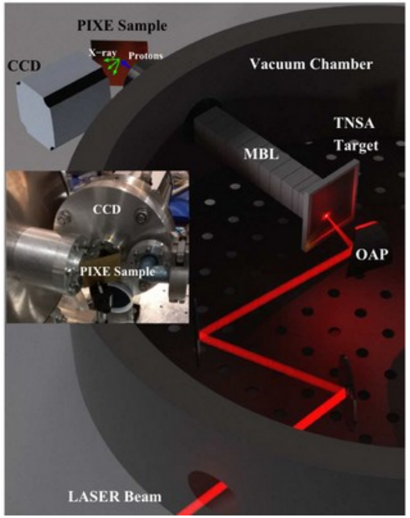
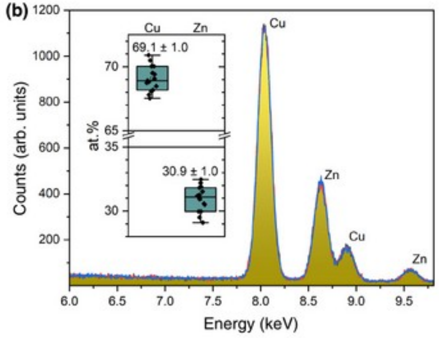


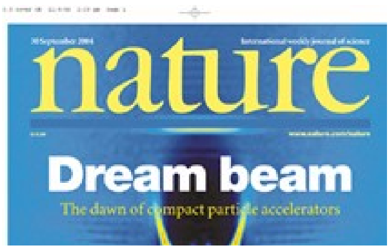


Motivations (2): Laser WakeField Acceleration (of electrons)

Electrons can be accelerated up to relativistic energy (several 100s MeV up to ~10GeV) via the so-called Laser WakeField Acceleration (LWFA) process

A "historical" taste of the literature on LWFA

1979: proposal by Tajima&Dawson



2004: "high quality" beams reported

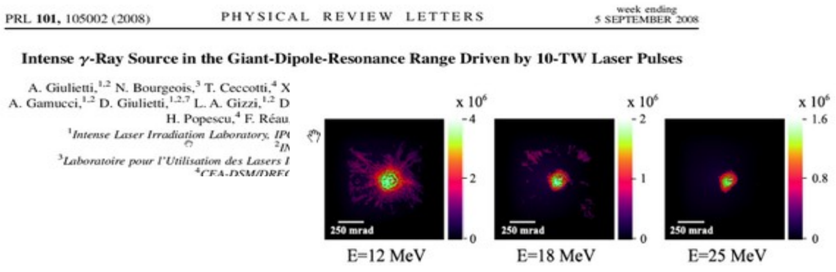
2006: GeV energy level achieved


LETTERS

GeV electron beams from a centimetre-scale accelerator

nature physics | VOL 2 | OCTOBER 2006 | www.nature.com/naturephysics

Nowadays: routine production of stable e- bunches, Secondary X/gamma-ray sources, energy increase up to the ~10GeV level


PRL 101, 105002 (2008) PHYSICAL REVIEW LETTERS week ending 5 SEPTEMBER 2008

Intense γ-Ray Source in the Giant-Dipole-Resonance Range Driven by 10-TW Laser Pulses

A. Giulietti,[1,2] N. Bourgeois,[3] T. Ceccotti,[4] X
A. Gamucci,[1,2] D. Giulietti,[1,2,7] L. A. Gizzi,[1,2] D
H. Popescu,[4] F. Réau
[1]Intense Laser Irradiation Laboratory, IP
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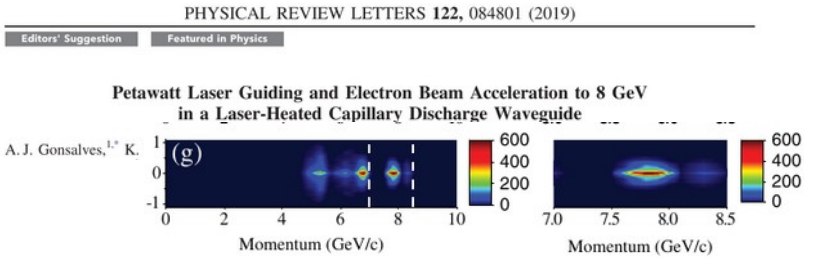

PHYSICAL REVIEW LETTERS 122, 084801 (2019)

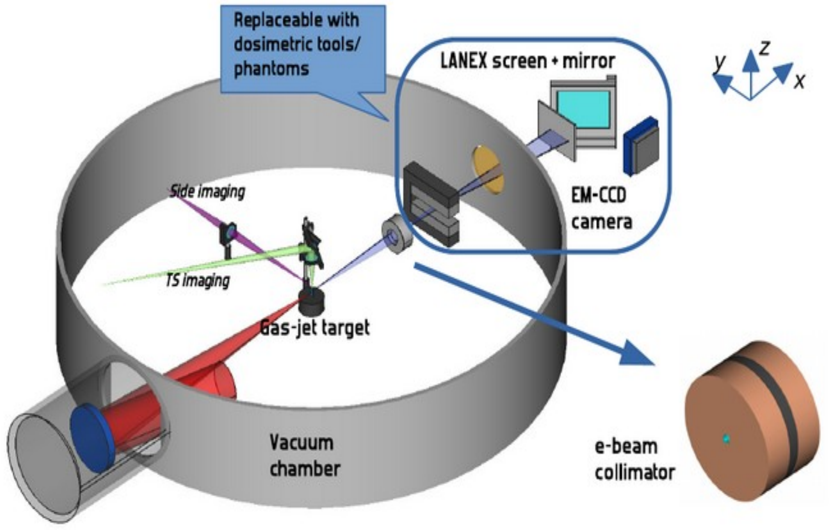
Editors' Suggestion Featured in Physics

Petawatt Laser Guiding and Electron Beam Acceleration to 8 GeV in a Laser-Heated Capillary Discharge Waveguide

A. J. Gonsalves,[1,*] K.

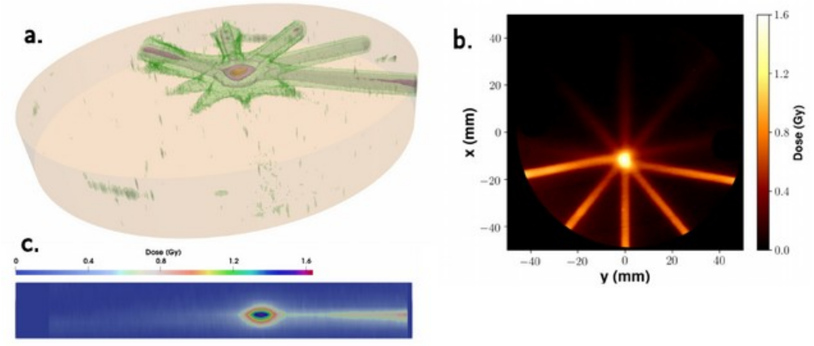
Motivations (3). Very High Energy Electrons (VHEE) for radiotherapy

Electrons with energy in the range ~100-250 MeV (so-called Very High Energy Electrons) are particularly promising for novel protocols/modalities in radiotherapy Recent experiments aimed at demonstrating the feasibility of advanced irradiation modalities (similar to current photon based radiotherapy) with laser-driven VHEE pencil beams

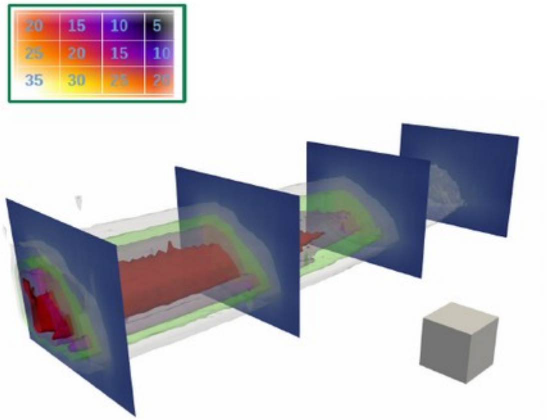


L. Labate *et al.*, Sci. Rep. **10**, 17307 (2020)

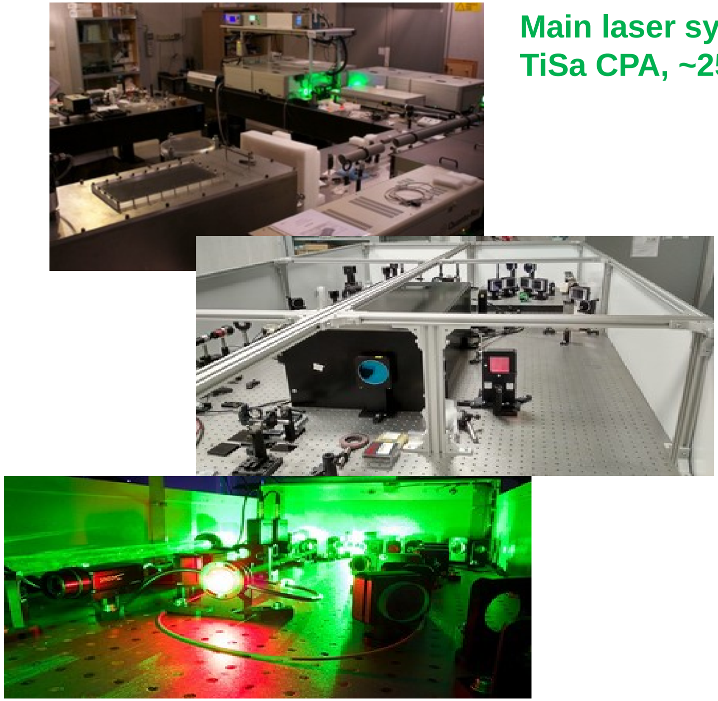
Multi-field irradiation



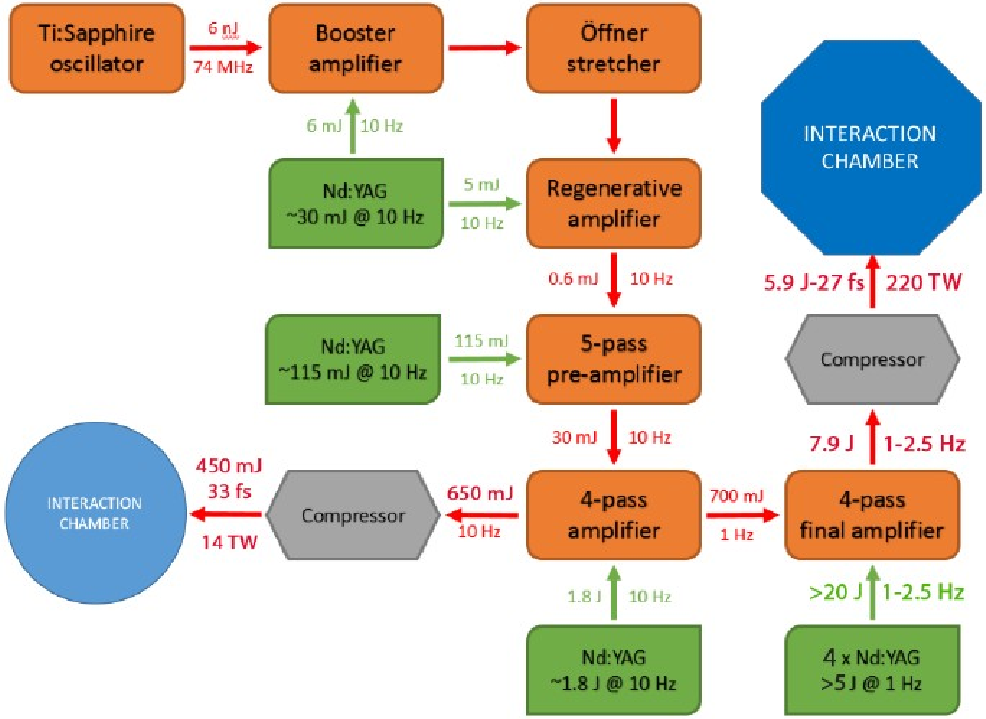
Intensity modulation



A 100TW class ultrashort laser system: The ILIL laser



Main laser system (until may 2025):
TiSa CPA, ~25fs, >5J energy (~220TW peak power), 1Hz rep rate



Bandwidth of a typical ultrashort laser

Uncertainty relation (between the w-t conjugated variables):

$$\delta t \, \delta\omega \geq \frac{1}{2}$$

Expressing it in terms of a wavelength bandwidth:

$$\delta\lambda \geq \frac{1}{2} \frac{\lambda^2}{2\pi c} \frac{1}{\delta t}$$

For a *Fourier-transform limited* pulse (we'll see what this does mean later)

$$\delta\lambda \sim \frac{\lambda^2}{c} \frac{1}{\delta t}$$

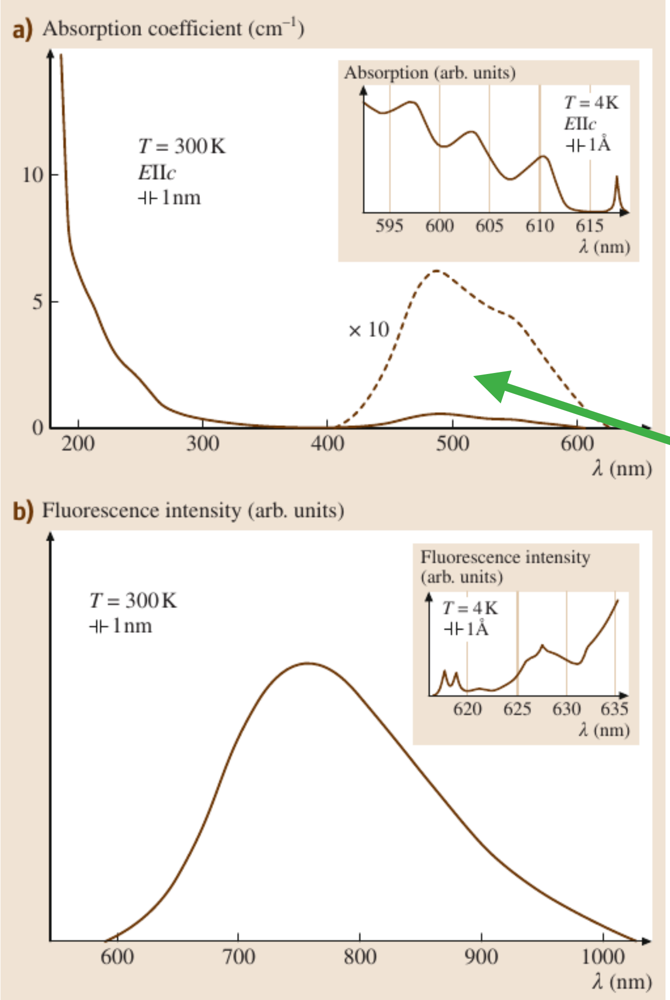
$$\omega = \frac{2\pi c}{\lambda}$$

$$\delta\omega = \left| \frac{\partial\omega}{\partial\lambda} \right| \delta\lambda = \frac{2\pi c}{\lambda^2} \delta\lambda$$

Putting some numbers in, we find that ~30fs laser pulses require (at least) a ~30nm bandwidth

The Ti:sapphire active medium

Index of refraction	1.76
Absorption cross section	6.5×10^{-20} cm^2 ($E \parallel c$)
Fluorescence lifetime	3.2 μs
Fluorescence bandwidth (FWHM)	≈ 200 nm
Peak emission wavelength	790 nm
Peak stimulated emission cross section	4.1×10^{-19} cm^2 ($E \parallel c$)
	2.0×10^{-19} cm^2 ($E \perp c$)
Quantum efficiency	≈ 0.9–1
Saturation fluence	0.9 J/cm^2
Dopant concentration	0.1% (weight)
Growth	Czochralski, heat exchange
T_m	2050 °C
Thermal conductivity	28 W/(m K)
Thermal lens (dn/dT)	12×10^{-19} K^{-1}



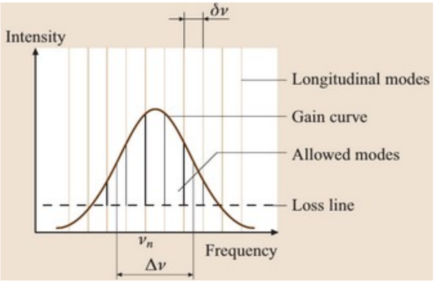
Optical pumping using 2nd harmonic laser radiation from Nd Q-switched solid-state lasers

Ultrashort laser oscillator: KLM

A quick recap on mode-locking

Laser oscillator: optical cavity + active medium (providing amplification when the population inversion is established by a pumping process)
Optical cavity: longitudinal modes with regularly spaced frequencies:

$$\nu_k = k\left(\frac{c}{2L}\right) \qquad \delta\nu = \nu_{n+1} - \nu_n = \frac{c}{2L} = \frac{1}{T_{RT}}$$



The resulting electric field at a given point (for instance, at the output mirror) can be written as

$$E(t) = \sum_{n=0}^{N-1} E_n \sin[2\pi(\nu_0 + n\delta\nu)t + \varphi_n(t)]$$

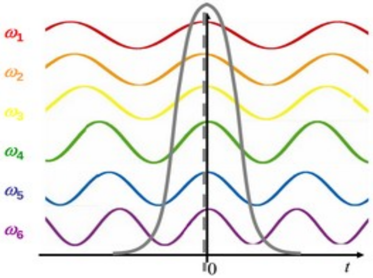
When the relative phase is equal to zero:
1. Maxima of the resulting "pulse" repeat in time with a period

$$\delta t = \frac{2\pi}{\delta\omega}$$

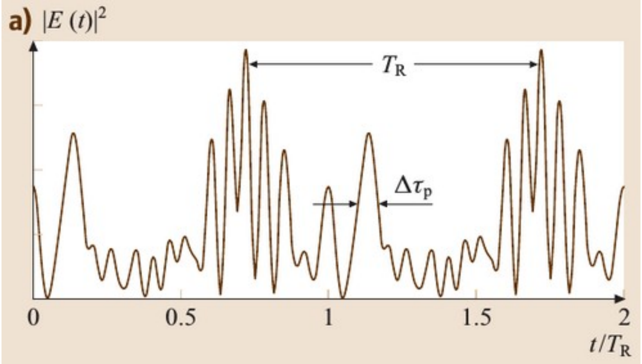
2. The higher number of "modes" we consider, the shorter the resulting maxima

What does it happen when I consider a non-null (random) phase?

"Locked" phases



Random phases



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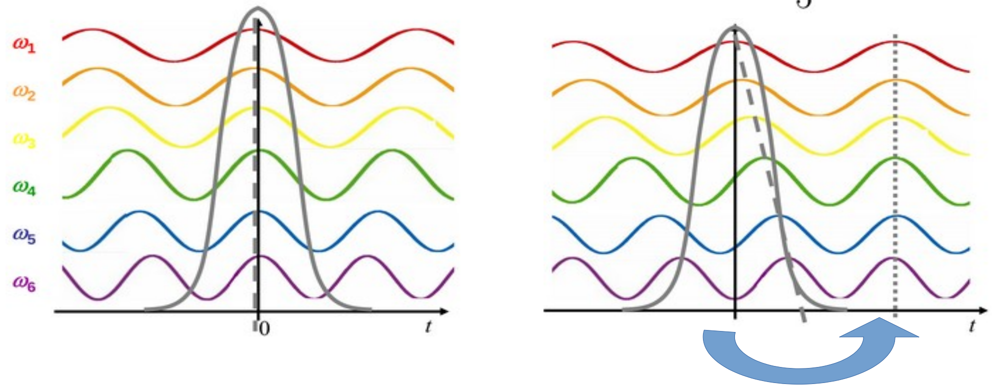
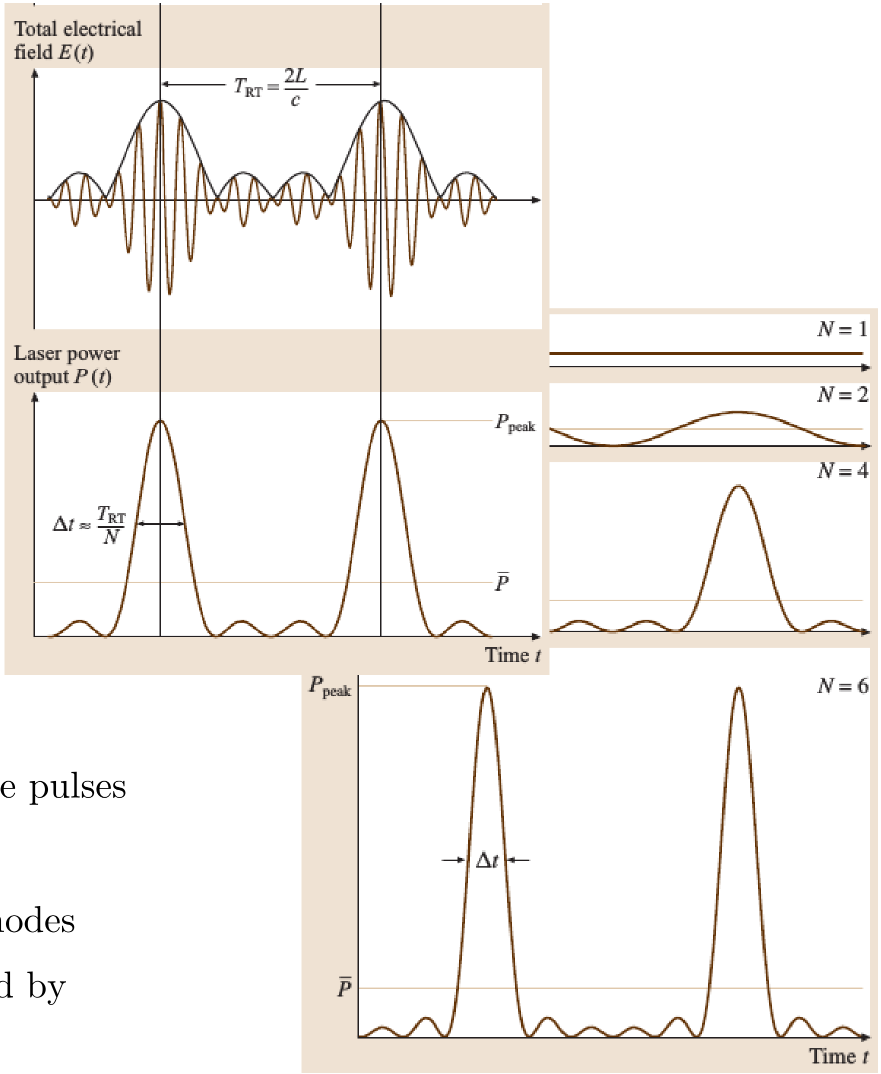
Constant or linear dependence of the phase on the frequency

$\phi(\omega_i) = 0$ $\qquad\qquad$ $\phi(\omega_i) = (i-1)\dfrac{\pi}{5}$





It can be easily shown that, for $\varphi_n = n\alpha$

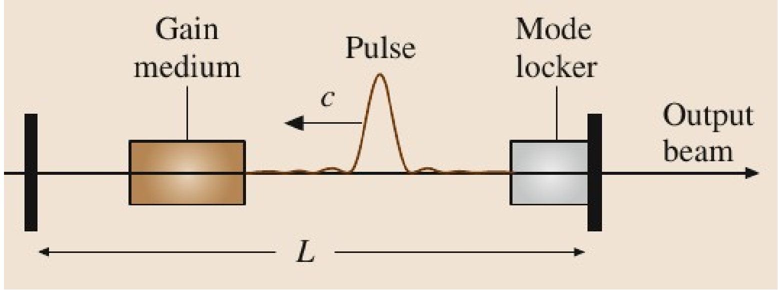
$$E(t) = E_0 \sin\left[2\pi\left(\nu_0 + \frac{N-1}{2}\delta\nu\right)t\right]\frac{\sin(N\pi\delta\nu t)}{\sin(\pi\delta\nu t)}$$

1. Power emitted as a pulse train, with a time separation between the pulses corresponding to the cavity round-trip
2. The peak power grows as the *square* of the number of modes
3. The FWHM of each pulse decreases linearly with the number of modes

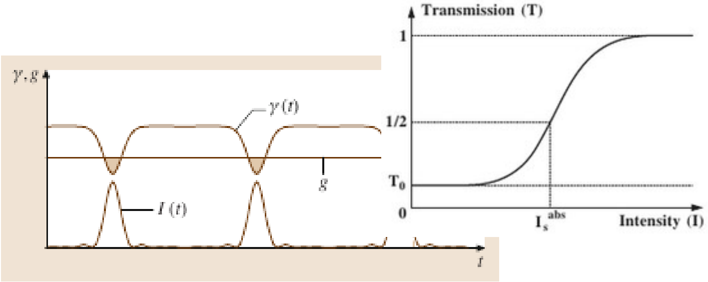
From a practical viewpoint, achieving a mode-locking is accomplished by inducing a periodic modulation of the gain of the cavity

KLM oscillators of high power lasers

A physical process is used, able to modulate losses in the cavity, which depends only on the instantaneous intensity of the laser (i.e., it doesn't depend on any external source)



Mode-locking oscillator

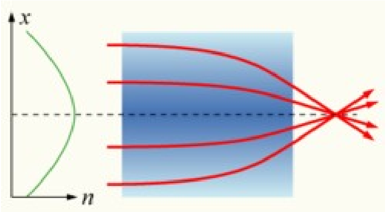


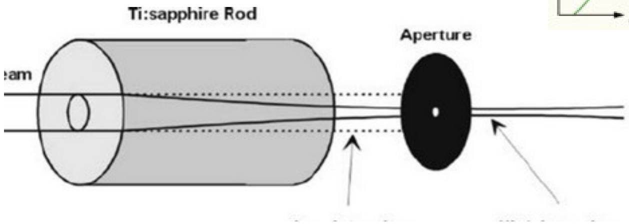
→ example: *saturable absorber*

Optical Kerr effect

$$n = n_0 + n_2 I$$

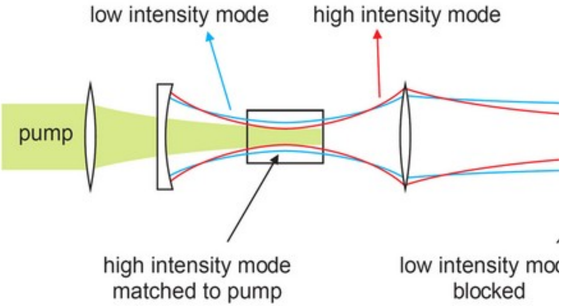
→ it leads to a (self) *focusing* of a beam with a *transversely non-constant* intensity



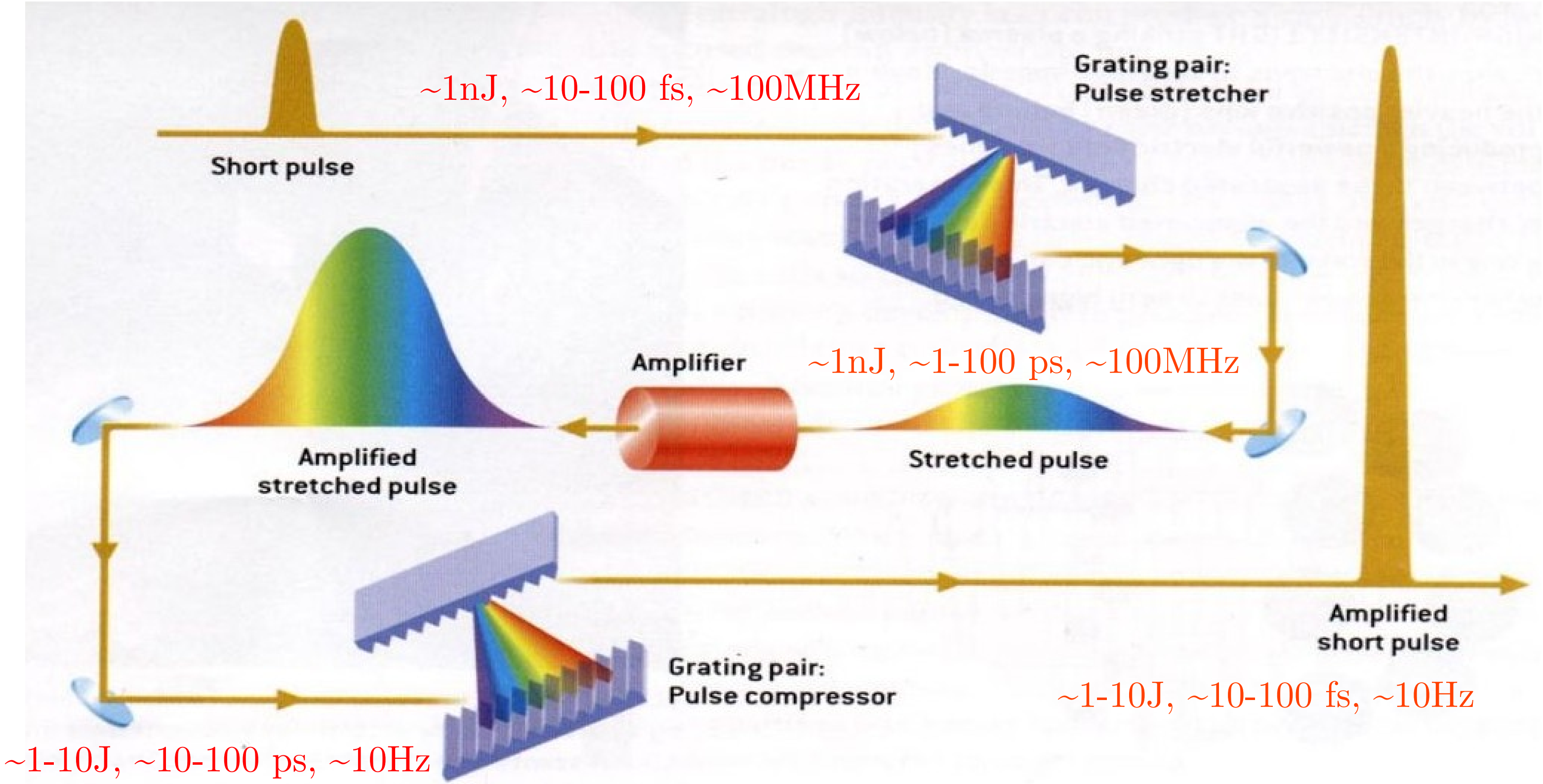


The "aperture" (iris) is not needed: overlapping between signal and pump acts to modulate the gain

Optical scheme of a KLM oscillator

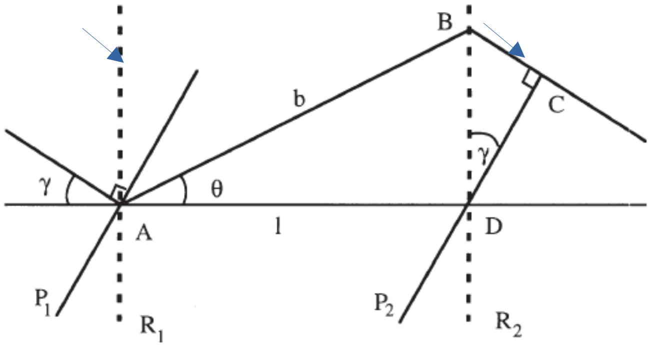


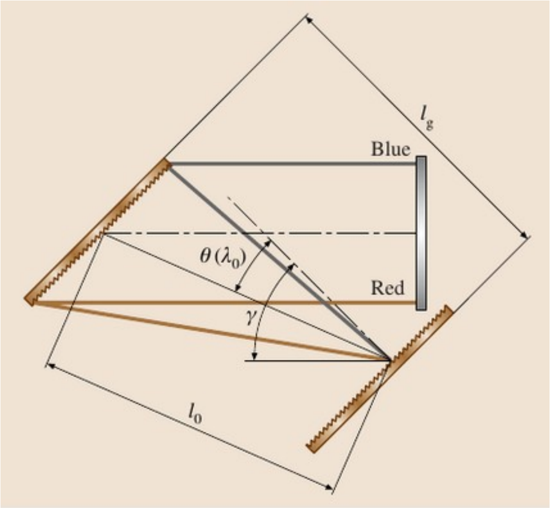
Amplifying ultrashort pulse: The Chirped Pulse Amplification (CPA) technique



A grating stretcher (or compressor): How it works

Diffraction grating(s) with pitch *d*



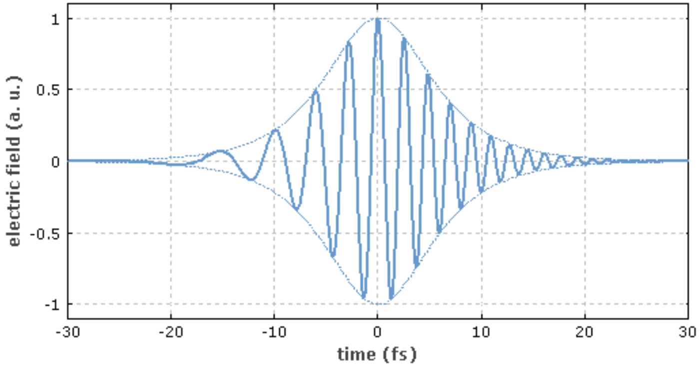


For each grating

$$d(\sin\gamma + \sin\theta) = \lambda$$

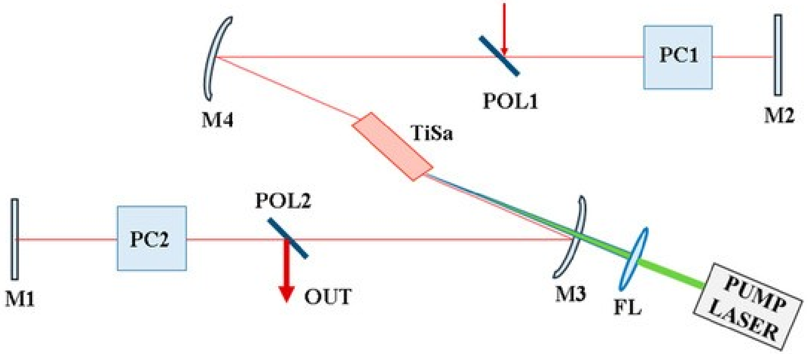
$$t = L/c = (\mathrm{AB} + \mathrm{BC})/c.$$

Thus, the optical path “spent” in the system depends upon the wavelength. At the exit, I have a longer and *chirped* pulse



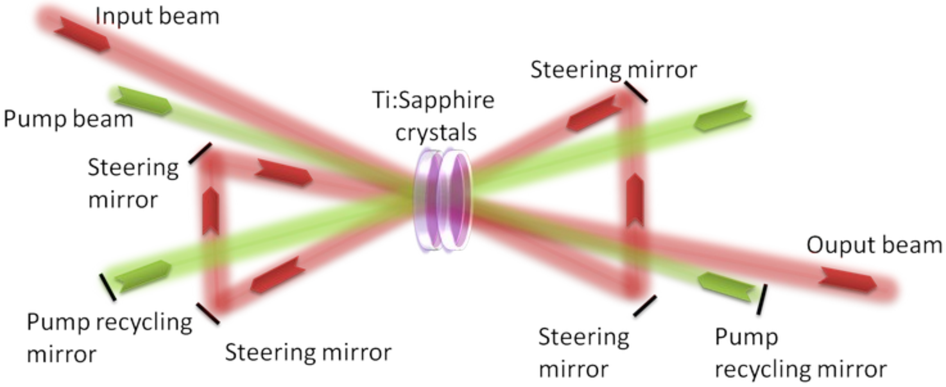
Amplifying (ultra)short pulses: Amplification stages

"Regenerative" amplifier



TiSa, Brewster angle cut Ti:sapphire crystal
M1-M4, high reflectance mirrors at 800 nm
PC1, injection Pockels Cell
PC2, extraction Pockels Cell
POL 1, POL 2, Polarizers
FL, Focusing lens

"Multi-pass" amplifier



Toward the characterization of the pulse: time (longitudinal) description of an ultrashort pulse

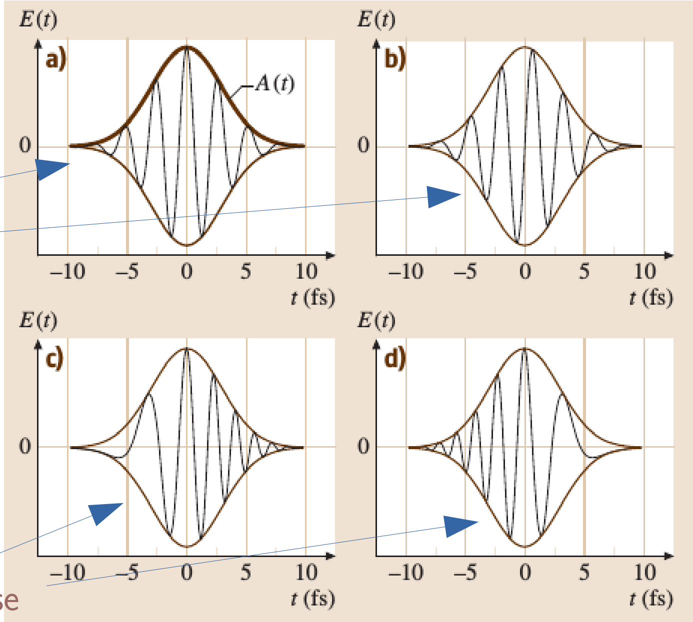
At a fixed point in space, for a linearly polarized pulse, the electric field can be simply written as $E(t) = A(t)\cos(\Phi_0 + \omega_0 t)$

The field envelope and the pulse intensity are related by the expression (A in V/m, I in W/cm2)

$$A(t) = \sqrt{\frac{2}{\varepsilon_0 c}}\sqrt{I(t)} = 27.4\sqrt{I(t)}$$

Φ_0 Absolute phase,
or *carrier envelope phase (CEP)*

Cosine pulse
Sine pulse



Adding a time dependent phase function results in a so-called *chirped* pulse

$$\Phi(t) = \Phi_0 + \omega_0 t + \Phi_a(t)$$

An *instantenous frequency* can be defined as

$$\omega(t) = \frac{d\Phi(t)}{dt} = \omega_0 + \frac{d\Phi_a(t)}{dt}$$

Upchirped pulse
Downchirped pulse

Introducing the spectral amplitude and phase

Using Fourier analysis, the field and its Fourier transform can be written as

$$E(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}(\omega) e^{i\omega t} \, d\omega \qquad \tilde{E}(\omega) = \int_{-\infty}^{\infty} E(t) e^{-i\omega t} \, dt$$

Of course, the knowledge of one of the two description is enough to completely characterize the pulse.

Most often, the so called "analytic signal" is used.

Being *E(t)* real, its Fourier transform is a Hermitian function: $\tilde{E}(\omega) = \tilde{E}^*(-\omega)$

This means that the knowledge of the Fourier transform for positive frequencies is enough to fully retrieve the signal

We can thus *define*, for convenience, a new function in the frequency domain, retaining only the positive part of the FT:

$$\tilde{E}^+(\omega) = \begin{cases} \tilde{E}(\omega) & \text{for } \omega \geq 0 \\ 0 & \text{for } \omega < 0 \end{cases}$$

and the corresponding FT^{-1}

$$E^+(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{E}^+(\omega) e^{i\omega t} \, d\omega$$

According to the above observation, $E^+(t)$, which is called *analytic signal*, is enough to retrieve the "real" field $E(t)$

Introducing the spectral amplitude and phase (2)

In the spectral domain we can introduce a *spectral amplitude* and a *spectral phase* as

$$\tilde{E}^{+}(\omega) = |\tilde{E}^{+}(\omega)|\mathrm{e}^{-i\phi(\omega)} \propto \sqrt{I(\omega)}\mathrm{e}^{-i\phi(\omega)}$$

What do the spectral amplitude and phase mean?

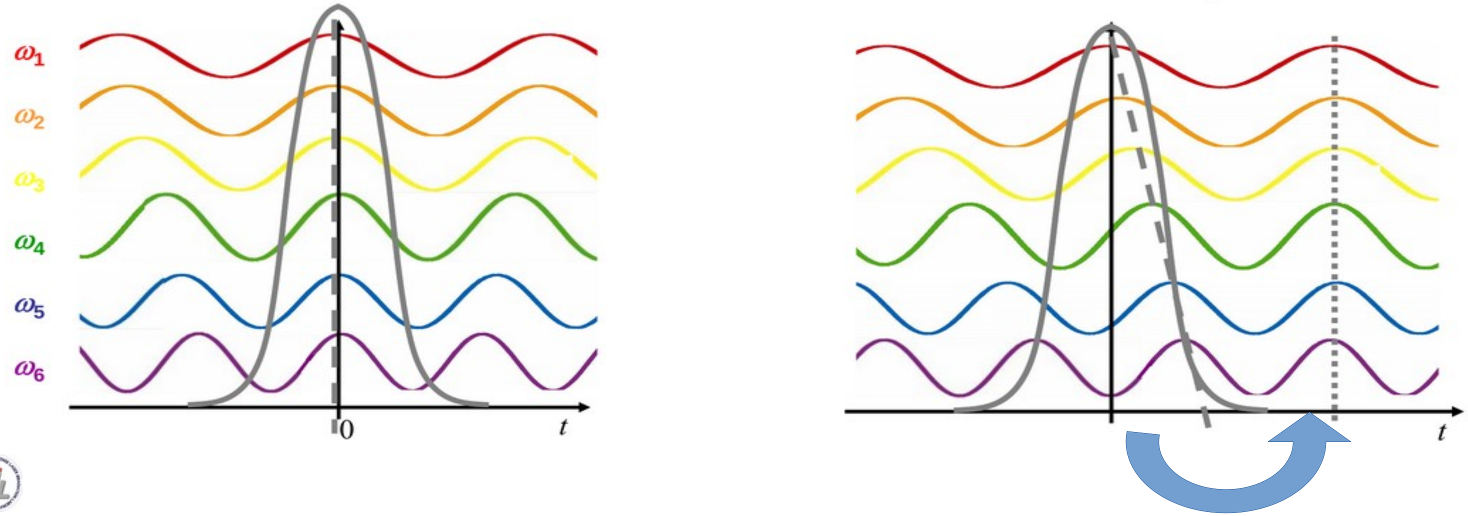
Spectral amplitude: proportional to the square root of *I*(ω), the usual "spectrum" as measured by a spectrometer

The **spectral phase** is basically the (relative) phase of each frequency in the waveform

What is the effect in the time domain?

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}(\omega)\,\mathrm{e}^{i\omega t}\,\mathrm{d}\omega$$

$\phi(\omega_i) = 0$ $\qquad\qquad$ $\phi(\omega_i) = (i-1)\frac{\pi}{5}$



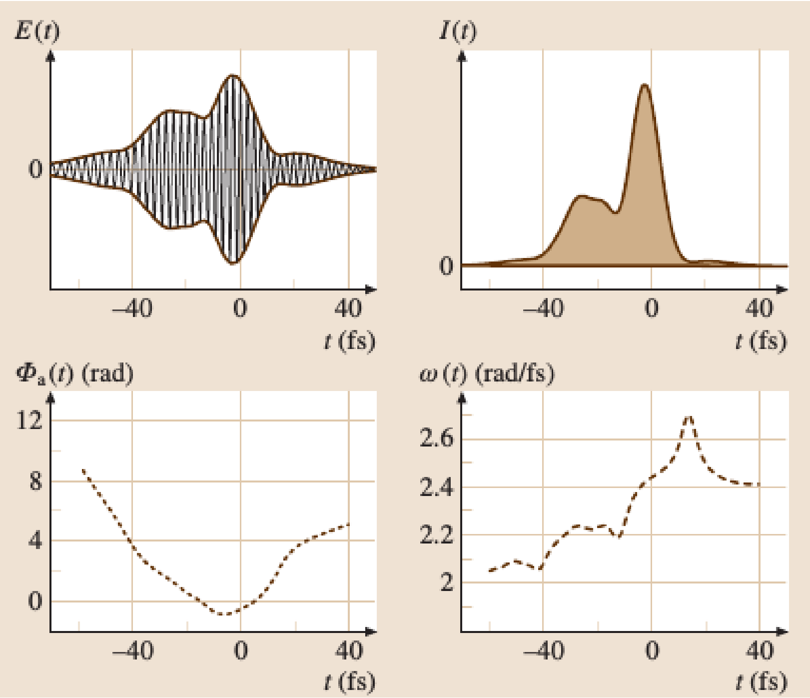
Time vs frequency space (behaviour)

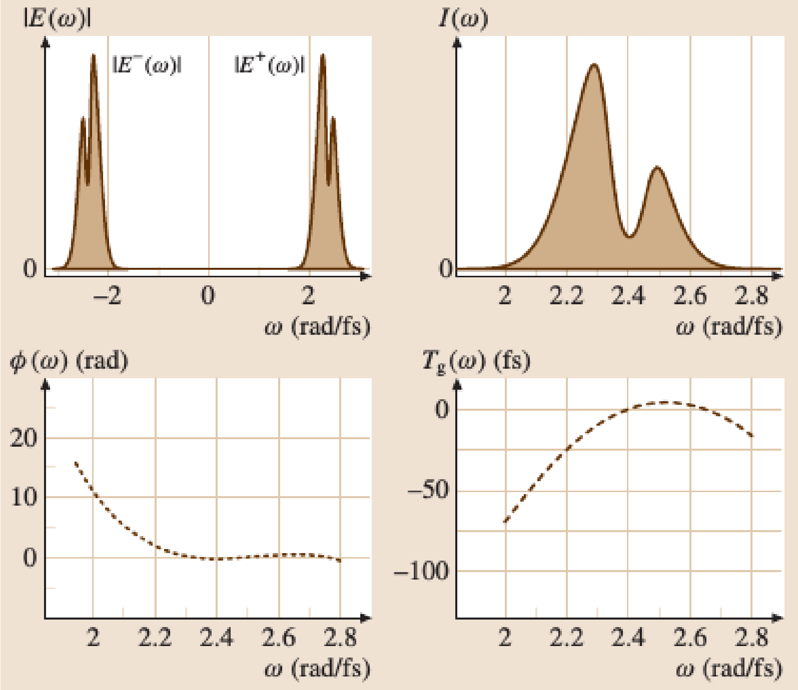
Time domain

$$E(t) = A(t)e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)}$$

Frequency

$$\tilde{E}^+(\omega) = |\tilde{E}^+(\omega)|e^{-i\phi(\omega)} \propto \sqrt{I(\omega)}e^{-i\phi(\omega)}$$





Spectral phase: Taylor expansion and the role of the different "orders"

In general, the spectral phase can be expanded into a Taylor series:

$$\phi(\omega) = \sum_{j=0}^{\infty} \frac{\phi^{(j)}(\omega_0)}{j!} \cdot (\omega - \omega_0)^j \qquad \text{where, of course,} \qquad \phi^{(j)}(\omega_0) = \left.\frac{\partial^j \phi(\omega)}{\partial \omega^j}\right|_{\omega_0}$$

This holds for a *well-defined* pulse. Basically, it means that each term in the expansion produces a pulse broadening or distortion that is significantly smaller than that of the previous term (see * for a deeper discussion on the optical meaning)

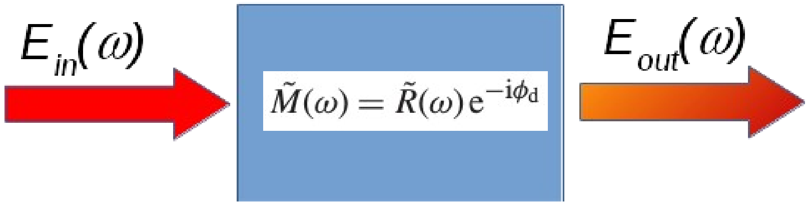
$$\left.\frac{\partial \phi}{\partial \omega}\right|_{\omega_0} (\omega - \omega_0) \gg \frac{1}{2!} \left.\frac{\partial^2 \phi}{\partial \omega^2}\right|_{\omega_0} (\omega - \omega_0)^2 \gg \frac{1}{3!} \left.\frac{\partial^2 \phi}{\partial \omega^2}\right|_{\omega_0} (\omega - \omega_0)^3 \gg \ldots$$

Terminology: 2nd order term $\rightarrow$ Group Velocity Dispersion (GVD), 3rd order term $\rightarrow$ Third Order Dispersion (TOD)

Why introducing the spectral amplitude and phase?

A linear optical system acts on an input field by a multiplication by a (complex) transfer function in the frequency domain:

$$\tilde{E}^{+}_{\text{out}}(\omega) = \tilde{M}(\omega)\tilde{E}^{+}_{\text{in}}(\omega) = \tilde{R}(\omega)\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{d}}}\tilde{E}^{+}_{\text{in}}(\omega)$$



The spectral phase of the output pulse is thus modified according to

$$\phi_{in}(\omega) \mapsto \phi_{in}(\omega) + \phi_d(\omega)$$

An initially unchirped pulse ($\phi''_{in}(\omega_0) = 0$) can acquire a chirp if $\phi''_d(\omega_0) \neq 0$

*see D.N. Fittinghoff *et al.*, IEEE J. Sel. Top. Quant. Electr. **4**, 430 (1998)

Spectral phase: the meaning of the first orders (exercise)

Spectral phase expansion $$\phi(\omega)=\sum_{j=0}^{+\infty}\frac{1}{j!}\phi^{(j)}(\omega_0)(\omega-\omega_0)^j$$

"Reference" pulse with $\phi(\omega)=0$, so that $\tilde{E}_{ref}(\omega)=\left|\tilde{E}_{ref}(\omega)\right|$ and $E^+_{ref}(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\tilde{E}_{ref}(\omega)\right|\mathrm{e}^{i\omega t}\,\mathrm{d}\omega$

$$\boxed{\phi(\omega)=\phi(\omega_0)}$$

$$\tilde{E}^+(\omega)=\left|\tilde{E}^+(\omega)\right|\mathrm{e}^{-i\phi(\omega_0)}=\left|\tilde{E}^+_{ref}(\omega)\right|\mathrm{e}^{-i\phi(\omega_0)}$$

On calculating the IFT, one gets

$$E^+(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\tilde{E}^+(\omega)\right|\mathrm{e}^{-i\phi(\omega_0)}\mathrm{e}^{i\omega t}\,\mathrm{d}\omega=\mathrm{e}^{-i\phi(\omega_0)}E^+_{ref}(t)$$

This corresponds to acquiring an absolute phase $\phi(\omega_0)$

$$\boxed{\phi(\omega)=\phi'(\omega_0)(\omega-\omega_0)}$$

Pulse with a 1st order term

$$\tilde{E}^+(\omega)=\left|\tilde{E}_{ref}(\omega)\right|\mathrm{e}^{-i\phi'(\omega_0)(\omega-\omega_0)}$$

On calculating the IFT, one gets

$$E^+(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|\tilde{E}^+_{ref}(\omega)\right|\mathrm{e}^{-i\phi'(\omega_0)(\omega-\omega_0)}\mathrm{e}^{i\omega t}\,\mathrm{d}\omega=\mathrm{e}^{i\phi'(\omega_0)\omega_0}\tilde{E}^+(t-\phi'(\omega_0))$$

This corresponds to a time shift of the pulse, with

$$T_g=\phi'(\omega_0)$$

→ To summarize: constant and linear terms in the spectral phase have no effects on the pulse duration

Spectral phase: High order terms and pulse duration

For a pulse with a given bandwidth (and spectrum), the shortest duration is reached when no chirp occurs; in the frequency domain, this translates into the spectral phase exhibiting a constant or linear dependence upon ω

We start calculating the pulse duration for a general pulse as

$$\Delta t^2 = \int_{-\infty}^{+\infty} (t - \langle t \rangle)^2 I(t)\,\mathrm{d}t = \int_{-\infty}^{+\infty} |(t - \langle t \rangle) E(t)|^2\,\mathrm{d}t$$

$$\mathcal{F}\left[(t - \langle t \rangle) E(t)\right] = i e^{-i\omega\langle t \rangle} \frac{\partial}{\partial\omega}\left(\mathrm{e}^{i\omega\langle t \rangle}\tilde{E}(\omega)\right)$$

(easy to demonstrate)

Using the Plancherel's identity and the equation aside

$$= \int_{-\infty}^{+\infty} |\mathcal{F}\left[(t - \langle t \rangle) E(t)\right]|^2\,\mathrm{d}\omega = \int_{-\infty}^{+\infty} \left|\frac{\partial}{\partial\omega}\left(\mathrm{e}^{i\omega\langle t \rangle}\tilde{E}(\omega)\right)\right|^2\,\mathrm{d}\omega$$

and finally, on introducing the spectral amplitude and phase and calculating the derivative

$$\Delta t^2 = \int_{-\infty}^{+\infty} \left|\frac{\partial}{\partial\omega}\left|\tilde{E}(\omega)\right|\right|^2\,\mathrm{d}\omega + \int_{-\infty}^{+\infty} \left|\tilde{E}(\omega)\right|^2 \left|\frac{\partial}{\partial\omega}\left(\omega\langle t \rangle - \phi(\omega)\right)\right|^2\,\mathrm{d}\omega$$

The first integral is ever positive and depends upon the spectral amplitude (or, the spectrum). As for the second one:

$$\frac{\partial}{\partial\omega}\left(\omega\langle t \rangle - \phi(\omega)\right) = \langle t \rangle - \phi'(\omega_0) - \frac{\partial}{\partial\omega}(\text{spectral phase terms } O((\omega - \omega_0)^2))$$

We saw above that the second term accounts, in the time domain, for a pulse delay, $T_g = \phi'(\omega_0)$ so that the first two terms cancels out

Thus, a further (positive) contribution to the time duration exists if the spectral phase exhibits higher order terms (GVD, TOD, ...)

A pulse whose spectral phase only contains up to linear terms in w is called Fourier-transform limited: in the real domain, it features the smalles pulse duration compatible with the spectral width

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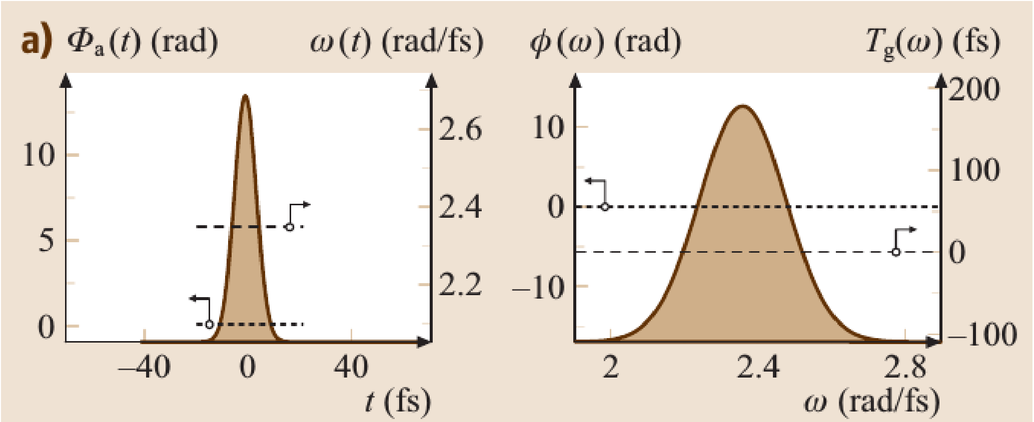
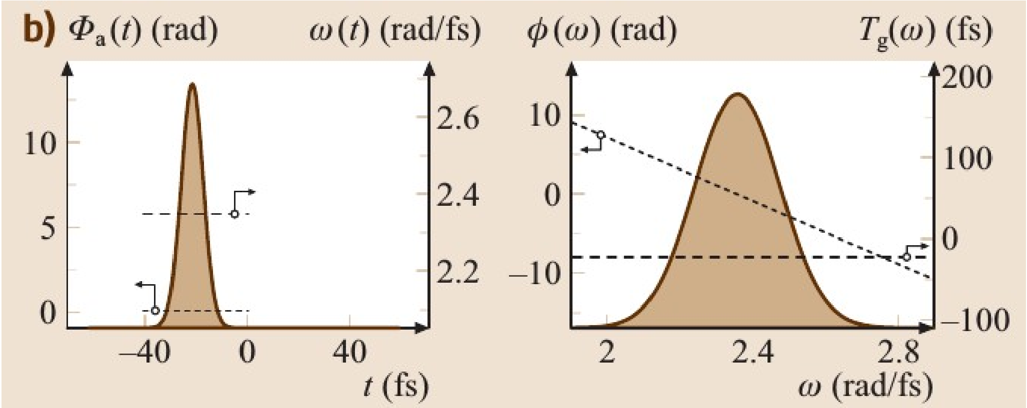
Spectral phase modifications and time duration: a few examples

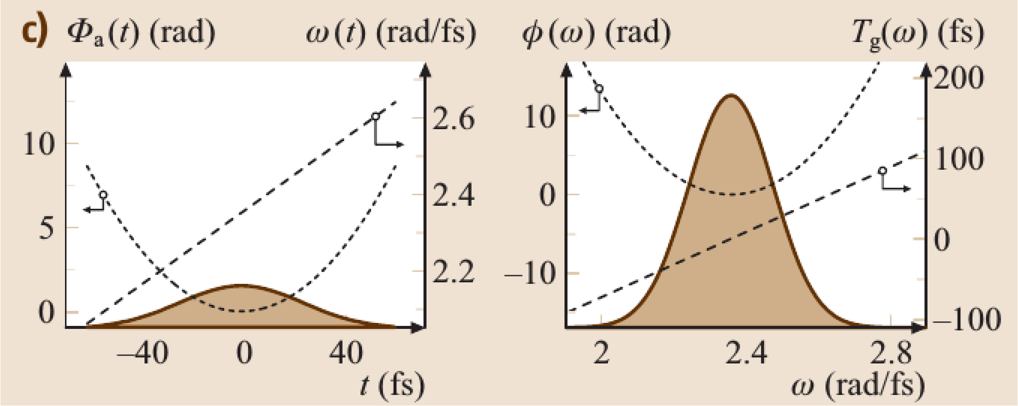


Unchirped (bandwidth limited) pulse, constant spectral phase

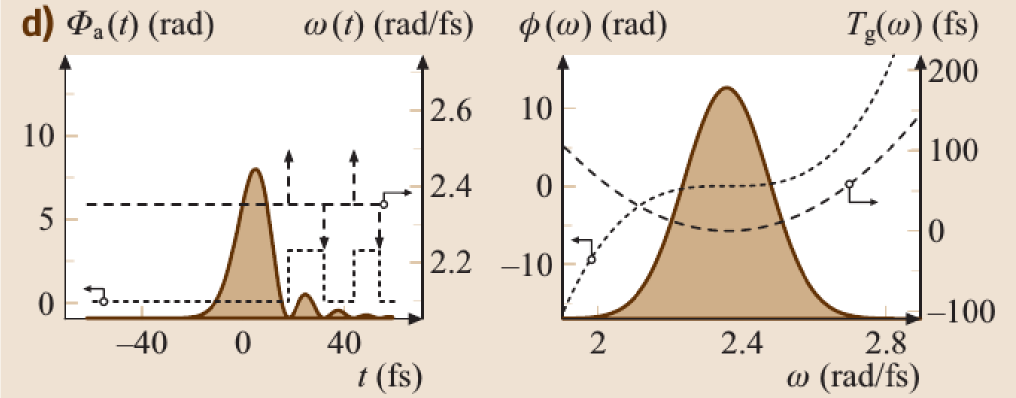


Unchirped pulse (bandwidth limited), shifted in time due to a linear (negative) spectral phase

Spectral phase modifications and time duration: a few examples (2)



Symmetrically broadened pulse, due to a 2nd order term in the spectral phase



3rd order spectral phase term, leading to quadratic group delay. In the time domain, oscillations appear before or after the main pulse, depending on the sign of the 3rd order

Spectral phase modification by common elements

Recall that

$$\tilde{E}^{+}_{\mathrm{out}}(\omega) = \tilde{M}(\omega)\tilde{E}^{+}_{\mathrm{in}}(\omega) = \tilde{R}(\omega)\,\mathrm{e}^{-\mathrm{i}\phi_{\mathrm{d}}}\,\tilde{E}^{+}_{\mathrm{in}}(\omega)$$

Transparent media

$$\phi_{\mathrm{m}}(\omega) = k(\omega)L = \frac{\omega}{c}n(\omega)L$$

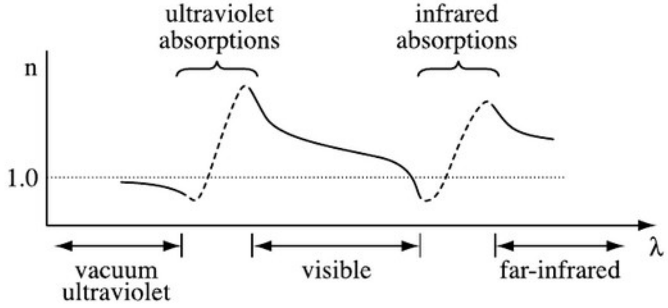
$$\frac{\mathrm{d}\phi_{\mathrm{m}}}{\mathrm{d}\omega} = \frac{L}{c}\left(n + \omega\frac{\mathrm{d}n}{\mathrm{d}\omega}\right) = \frac{L}{c}\left(n - \lambda\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right)$$

$$\phi''_{\mathrm{m}} = \frac{\mathrm{d}^2\phi_{\mathrm{m}}}{\mathrm{d}\omega^2} = \frac{L}{c}\left(2\frac{\mathrm{d}n}{\mathrm{d}\omega} + \omega\frac{\mathrm{d}^2 n}{\mathrm{d}\omega^2}\right)$$

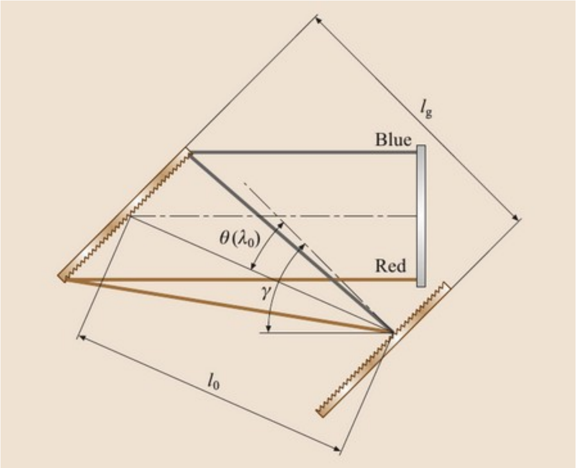
If $\mathrm{d}n/\mathrm{d}\lambda$ is not equal to zero (dispersion), each frequency will move with a different velocity and the pulse gets broadened (spectral phase wise, this results in a 2nd order not null)

Material	λ (nm)	$n(\lambda)$	$\frac{dn}{d\lambda}\cdot 10^{-2}$ $\left(\frac{1}{\mu m}\right)$	$\frac{d^2n}{d\lambda^2}\cdot 10^{-1}$ $\left(\frac{1}{\mu m^2}\right)$	$\frac{dn^3}{d\lambda^3}$ $\left(\frac{1}{\mu m^3}\right)$	T_g $\left(\frac{fs}{mm}\right)$	GDD $\left(\frac{fs^2}{mm}\right)$	TOD $\left(\frac{fs^3}{mm}\right)$
BK7	400	1.5308	−13.17	10.66	−12.21	5282	120.79	40.57
	500	1.5214	−6.58	3.92	−3.46	5185	86.87	32.34
	600	1.5163	−3.91	1.77	−1.29	5136	67.52	29.70
	800	1.5108	−1.97	0.48	−0.29	5092	43.96	31.90
	1000	1.5075	−1.40	0.15	−0.09	5075	26.93	42.88
	1200	1.5049	−1.23	0.03	−0.04	5069	10.43	66.12
SF10	400	1.7783	−52.02	59.44	−101.56	6626	673.68	548.50
	500	1.7432	−20.89	15.55	−16.81	6163	344.19	219.81
	600	1.7267	−11.00	6.12	−4.98	5980	233.91	140.82
	800	1.7112	−4.55	1.58	−0.91	5830	143.38	97.26
	1000	1.7038	−2.62	0.56	−0.27	5771	99.42	92.79
	1200	1.6992	−1.88	0.22	−0.10	5743	68.59	107.51
Sapphire	400	1.7866	−17.20	13.55	−15.05	6189	153.62	47.03
	500	1.7743	−8.72	5.10	−4.42	6064	112.98	39.98
	600	1.7676	−5.23	2.32	−1.68	6001	88.65	37.97
	800	1.7602	−2.68	0.64	−0.38	5943	58.00	42.19
	1000	1.7557	−1.92	0.20	−0.12	5921	35.33	57.22
	1200	1.7522	−1.70	0.04	−0.05	5913	13.40	87.30

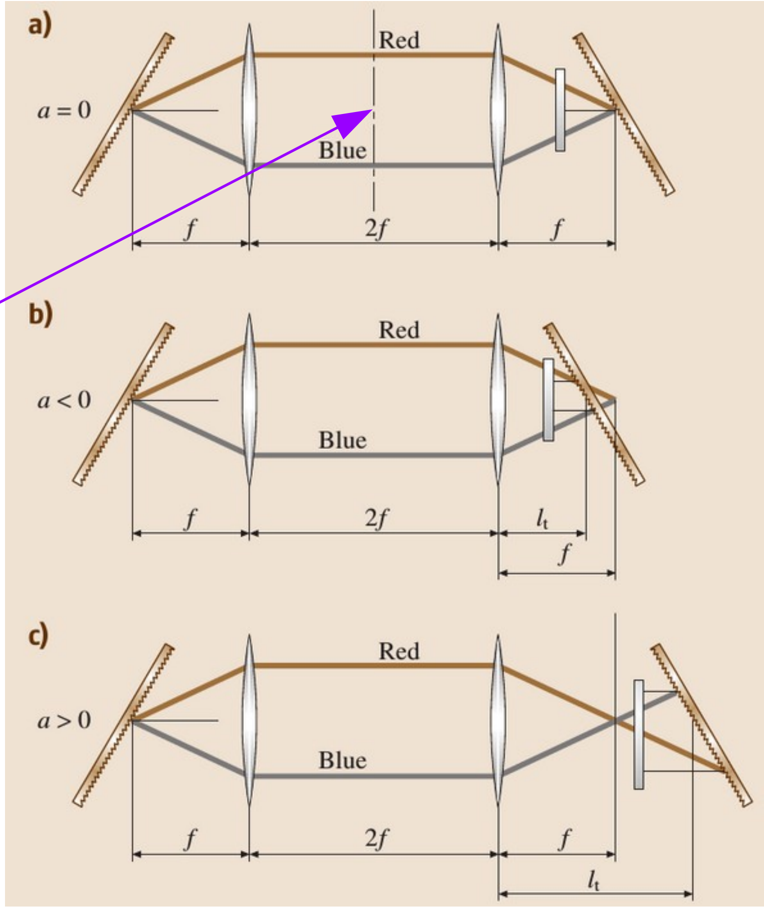
For ordinary transparent media in the visible region, normal dispersion is encountered ($\mathrm{d}n/\mathrm{d}\lambda>0$), which results in positive chirp (lower wavelengths arrive before higher ones)

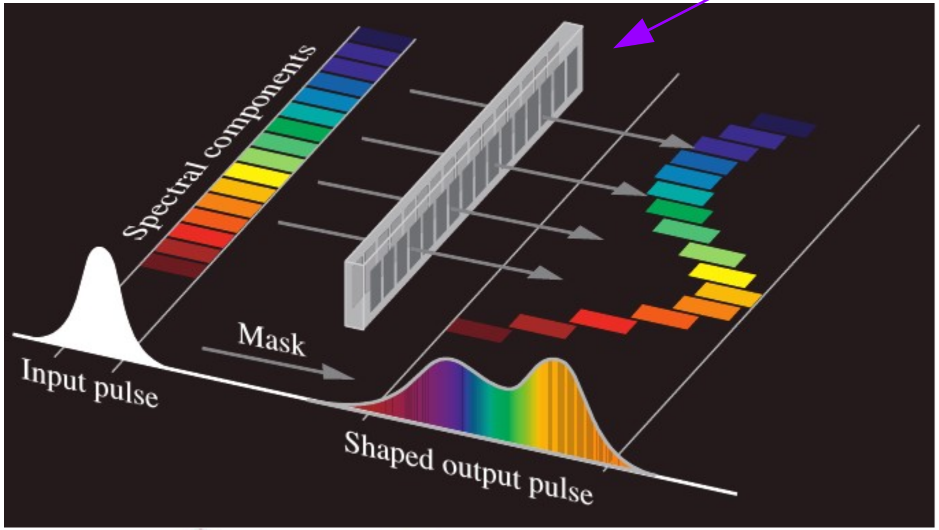


Modifying the spectral phase: stretcher/compressor and Spatial Light Modulators



The usage of grating elements to stretch or compress a pulse introduces 2nd order (and higher!) terms in the spectral phase, with both down (usually in a *compressor*) and up-chirping (usually in a *stretcher*)





For further details, see A.M. Weiner, *Rev. Sci. Instrum.* **71**, 1929 (2000)

"Recovering" the original pulse duration in CPA laser chains: Acousto-Optic Programmable Dispersive Filter

Each optical element in a CPA chain introduces a spectral phase modification. Recovering the original pulse duration means (ideally) removing all the "new" terms in the spectral phase to recover a FTL pulse

Amplitude and phase control of ultrashort pulses by use of an acousto-optic programmable dispersive filter: pulse compression and shaping

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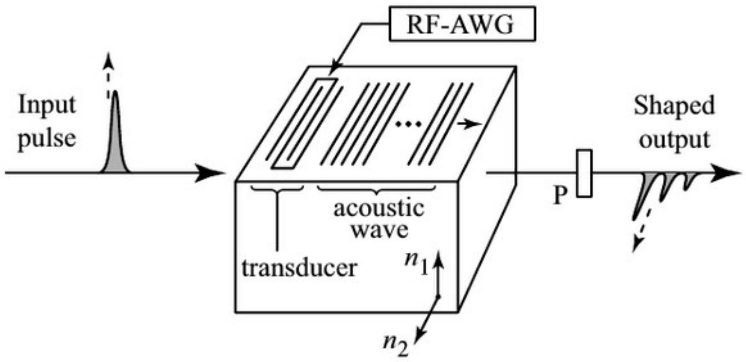
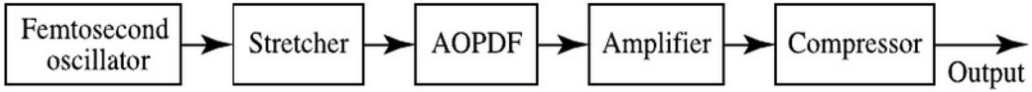
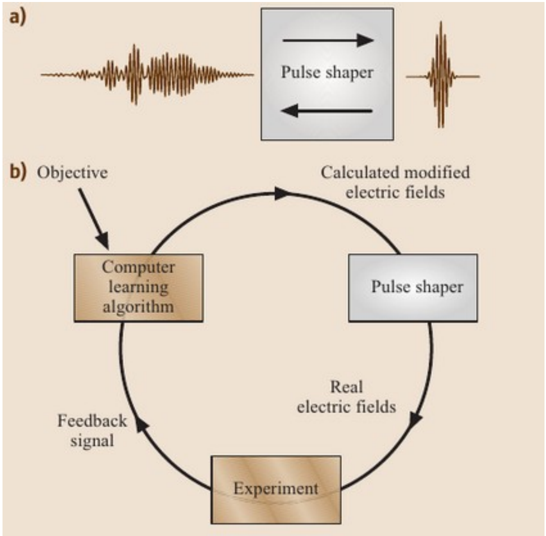
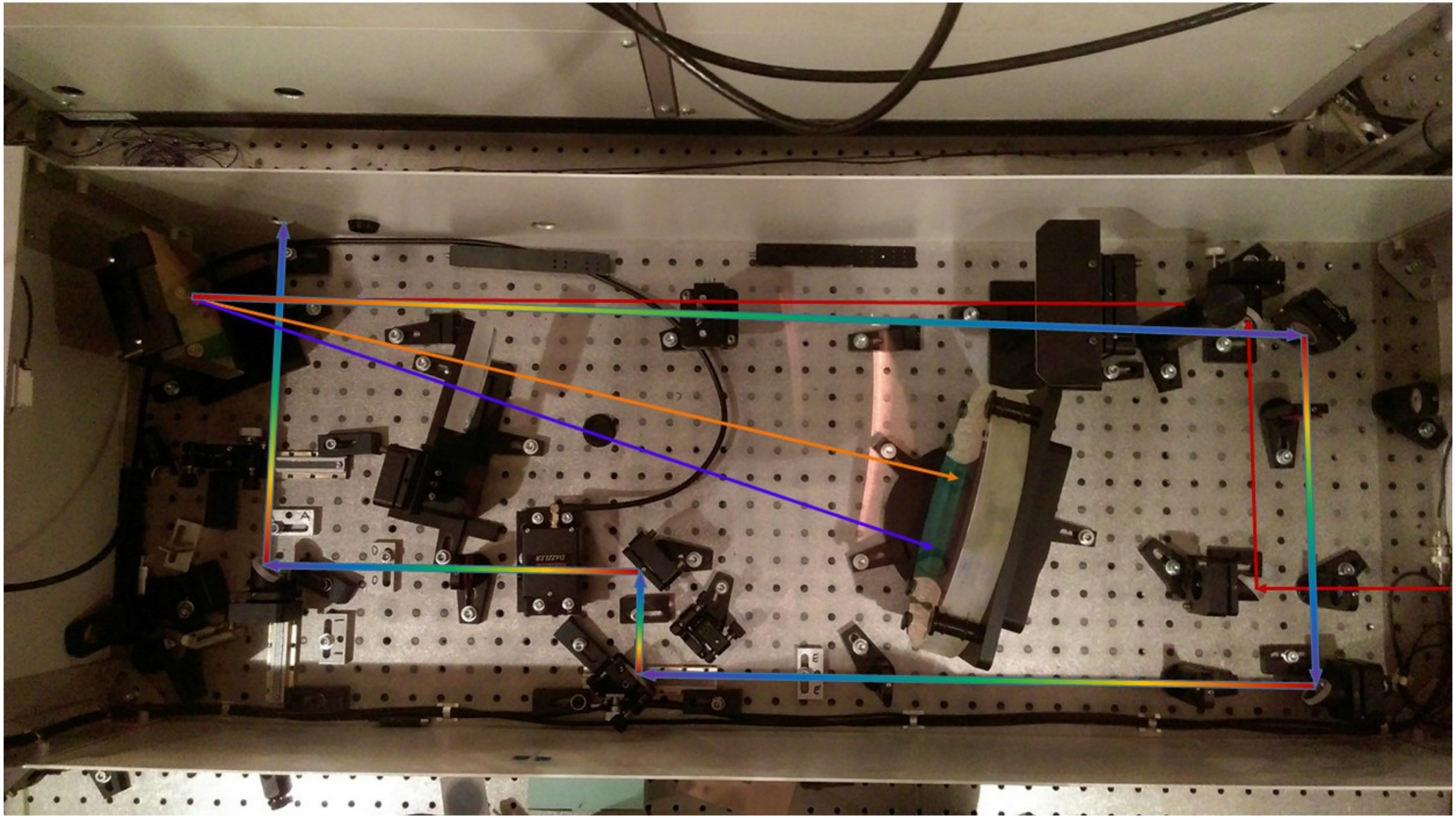


Acousto-optic filter (DAZZLER):
Pulse shaping is achieved via interaction of co-propagating acoustic and optical waves in a photoelastic medium
Acoustic wave produced by a (programmable) RF generator





A look at the DAZZLER in the ILIL 200TW laser



Measuring a pulse duration: Intro to autocorrelation methods

The duration of a pulse with a ~100fs duration cannot be measured using "electronic" methods (for instance, PD, streak-cameras, …). One has to resort to optical methods, working either in the time domain or in the frequency domain or combination of the two

Basic ingredients common to ALL the pulse measurement methods

1. Time-space transformation. A given delay is obtained by letting the pulse to be delayed travel longer paths; fs delays require (variable) micron-scale optical path lengths, which can be safely produced and measured using current technology translation stages and optical encoders
2. Use of the (auto)correlation functions to retrieve the pulse behaviour

Given two fields *Eref(t)* and *E(t)*, the measurement of their 1st order correlation function

$$G_1(\tau) = \int_{-\infty}^{+\infty} E^*_{ref}(t) E(t-\tau)\,\mathrm{d}t$$

allows *E(t)* to be recovered provided that $E_{ref}(t)$ (reference pulse) is fully known.

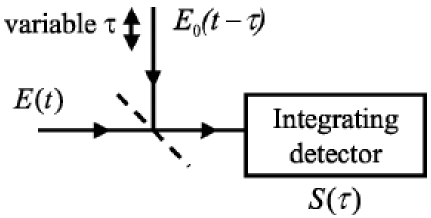
If a reference pulse is not available, more advanced methods must be employed

In what follows (and in the Martina's talk), detectors with response times much longer than the pulse duration are considered, so that basically they measure the pulse energy:

$$\text{Read signal} \propto \int_{-\infty}^{+\infty} |E(t)|^2\,\mathrm{d}t \propto \int_{-\infty}^{+\infty} I(t)\,\mathrm{d}t \propto \text{pulse energy}$$

Characterization of the temporal behaviour of a laser pulse using a reference pulse

Time-domain interferometry



A scan of a sufficiently large delay (>pulse duration) is carried out, and the signal corresponding to each delay is recorded

$$\begin{aligned} S(\tau) &\propto \int_{-\infty}^{+\infty} |E_{ref}(t-\tau)+E(t)|^2 \,\mathrm{d}t \\ &= \int_{-\infty}^{+\infty} |E_{ref}(t-\tau)|^2 \,\mathrm{d}t + \int_{-\infty}^{+\infty} |E(t)|^2 \,\mathrm{d}t + \left(\int_{-\infty}^{+\infty} E(t)E^*_{ref}(t-\tau)\,\mathrm{d}t + \text{c.c.}\right) \end{aligned}$$

Notice that the last two terms correspond to the 1st order correlation function.

Taking the Fourier Transform, one gets

$$\mathcal{F}(S)(\omega) = A\delta(\omega) + \tilde{E}(\omega)\tilde{E}^*_{ref}(\omega) + \tilde{E}(-\omega)\tilde{E}_{ref}(-\omega)$$

from which the spectral phase of the pulse can be retrieved provided that the reference pulse is completely characterized

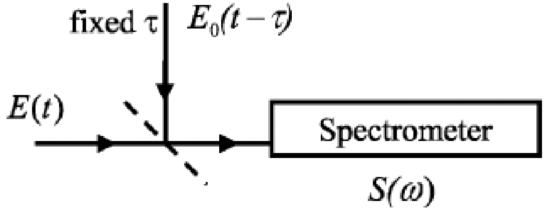
Recall that the spectral amplitude is simply related to the spectrum

$$\tilde{E}^+(\omega) = |\tilde{E}^+(\omega)|\mathrm{e}^{-i\phi(\omega)}$$

see Ch. Dorrer, M. Joffre, C.R. Acad. Sci. Paris **2**, 1415 (2001)

Characterization of the temporal behaviour of a laser pulse using a reference pulse (2)

Frequency-domain interferometry



The delay is kept at a fixed value, and the spectrum of the overlapped pulses is measured

$$
\begin{aligned}
S(\omega) \;\propto\; & \left|\mathcal{F}(E_{ref}(t-\tau)+E(t))\right|^2 = \left|\tilde{E}_{ref}(\omega)e^{i\omega\tau}+\tilde{E}(\omega)\right|^2 \\
= \; & \left|\tilde{E}_{ref}(\omega)\right|^2 + \left|\tilde{E}(\omega)\right|^2 + \left(\left|\tilde{E}_{ref}(\omega)\tilde{E}^*(\omega)\right| e^{-i(\phi_{ref}(\omega)-\phi(\omega))}e^{i\omega\tau} + \text{c.c.}\right) \\
= \; & \left|\tilde{E}_{ref}(\omega)\right|^2 + \left|\tilde{E}(\omega)\right|^2 + 2\left|\tilde{E}_{ref}(\omega)\right|\left|\tilde{E}^*(\omega)\right| \cos\left[\omega\tau-(\phi)_{ref}(\omega)-\phi(\omega))\right]
\end{aligned}
$$

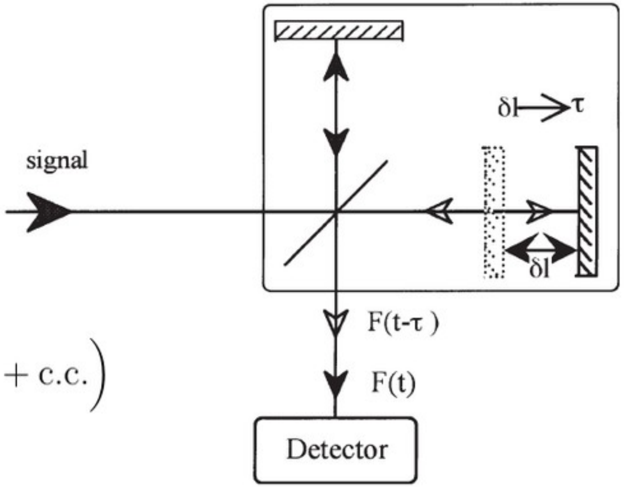
Interference fringes appear in the power spectrum with an average fringe spacing inversely proportional to the time delay

The phase of the fringe pattern yields the spectral phase difference between the reference and the unknown pulse

Main issue with these correlation techniques: a **completely** characterized reference pulse (with a spectrum larger than the one of the pulse to be measured) is ususally not available!

Characterization of the temporal behaviour of a laser pulse w/o a reference pulse: 1st order autocorrelation

Let's look at what happens when we have an interferometric *auto*-correlator: that is the pulse to be measured is splitted into two arms and recombined after being relatively time-delayed



$$
\begin{aligned}
S(\tau) &\propto \int_{-\infty}^{+\infty} |E(t-\tau)+E(t)|^2 \, \mathrm{d}t \\
&= \int_{-\infty}^{+\infty} |E(t-\tau)|^2 \, \mathrm{d}t + \int_{-\infty}^{+\infty} |E(t)|^2 \, \mathrm{d}t + \left(\int_{-\infty}^{+\infty} E(t)E^*(t-\tau) \, \mathrm{d}t + \mathrm{c.c.} \right) \\
&= 2\int_{-\infty}^{+\infty} |E(t)|^2 + \left(\int_{-\infty}^{+\infty} E(t)E^*(t-\tau) \, \mathrm{d}t + \mathrm{c.c.} \right)
\end{aligned}
$$

The terms in parenthesis correspond to the 1st order **auto**-correlation function.

Wiener-Khinchin theorem* $\mathcal{F}[G_1(\tau)] = \left|\tilde{E}(\omega)\right|^2$

Thus, taking the FT of the signal (with respect to the time delay between the two pulses), one ultimately only gets the power spectrum (Fourier-transform spectroscopy): no infos on the spectral phase

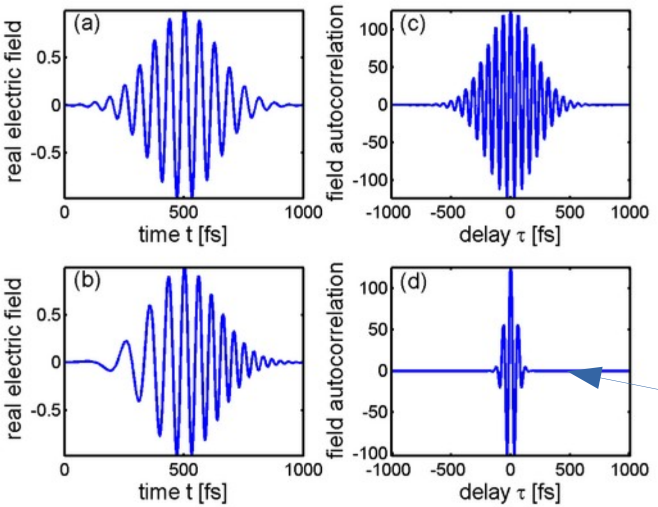
In fact, it can be shown that the full knowledge of *E(t)* requires the measurement of all the successive $G_n(t)$

Characterization of the temporal behaviour of a laser pulse w/o a reference pulse
1st order autocorrelation

Recalling that $E(t) = A(t)e^{i\Phi_0}e^{i\omega_0 t}e^{i\Phi_a(t)}$

$$\begin{aligned} S(\tau) &\propto 2\int_{-\infty}^{+\infty} |E(t)|^2 + \left(\int_{-\infty}^{+\infty} E(t)E^*(t-\tau)\,\mathrm{d}t + \mathrm{c.c.}\right) \\ &= 2\int_{-\infty}^{+\infty} |E(t)|^2\,\mathrm{d}t + 2\int_{-\infty}^{+\infty} A(t)A(t-\tau)\cos\left[\omega_0\tau + \Phi_a(t) - \Phi_a(t-\tau)\right]\,\mathrm{d}t \\ &\propto 1 + G_1(\tau) \end{aligned}$$

Unchirped pulse



Chirped pulse

The trace is symmetric, even in the presence of a chirp

Using only a Michelson interferometer: the width of S(t) is related to the coherence length of the pulse.

No way to recover the phase: 1st order autocorrelation is the IFT of the spectrum

Quite difficult to interpret, w/o any hint on the pulse chirp, spectrum, ...

With some assumption on both the pulse shape and phase (basically, no or negligible chirp), one can recover the pulse duration

Very basic method → more advanced methods in Martina's talk

Focusing an ultrashort, high power laser pulse: Why using reflective optics

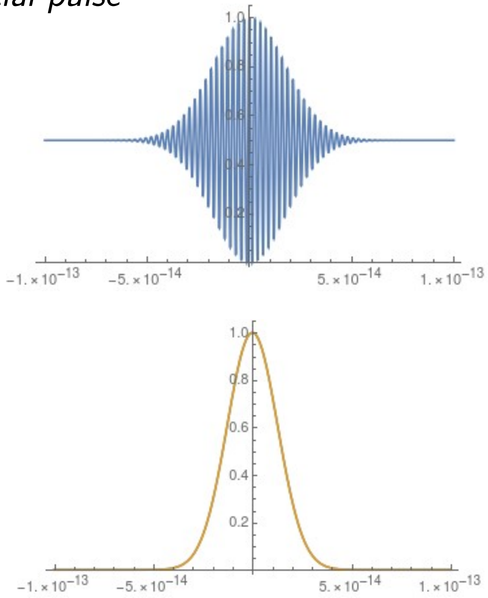
Ultrashort and high power laser pulse through a transparent medium (a lens):

1. Dispersive (linear) effects. As we have seen, an initial unchirped pulse (FTL) acquires high order terms in the spectral phase, leading to the pulse getting chirped (and thus stretched in time)

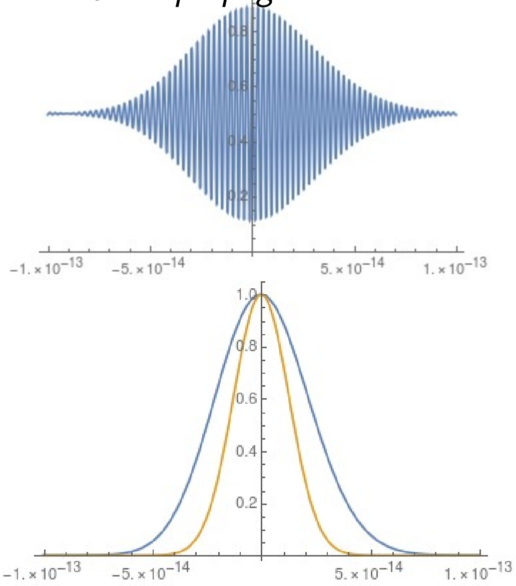
~~2. Nonlinear effects~~

A numerical example: a lens made up by a BK7 glass

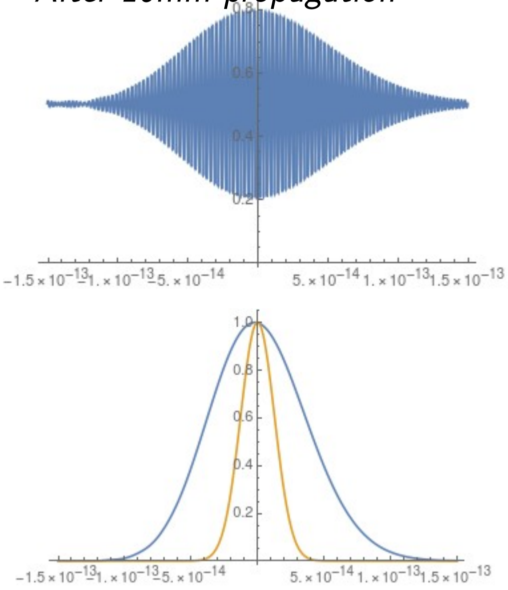
Initial pulse



After 5mm propagation



After 10mm propagation



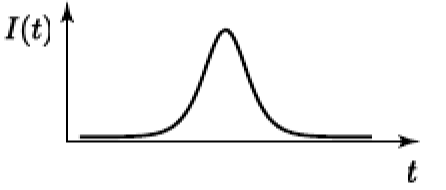
Focusing an ultrashort, high power laser pulse: Why using reflective optics

Ultrashort and high power laser pulse through a transparent medium (a lens):

1. ~~Dispersive (linear) effects. As we have seen, an initial unchirped pulse (FTL) acquires high order terms in the spectral phase, leading to the pulse getting chirped (and thus stretched in time)~~
2. Nonlinear effects

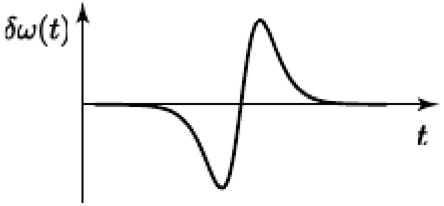
Gaussian pulse propagating in a medium with a 3rd order nonlinearity

$$n(t) = n_0 + n_2 I(t)$$



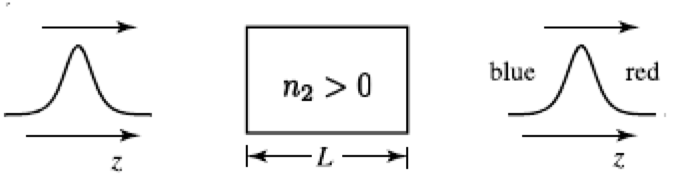
The 1st term accounts for the usual phase shift after a propagation over a length L.
The 2nd term leads to another contribution to the phase shift:

$$\phi_{\mathrm{NL}}(t) = -n_2 I(t) \omega_0 L / c.$$



According to what we said earlier, the instantaneous frequency acquires a time-dependent term:

$$\omega(t) = \omega_0 + \delta\omega(t) \qquad \delta\omega(t) = \frac{d}{dt}\phi_{\mathrm{NL}}(t)$$

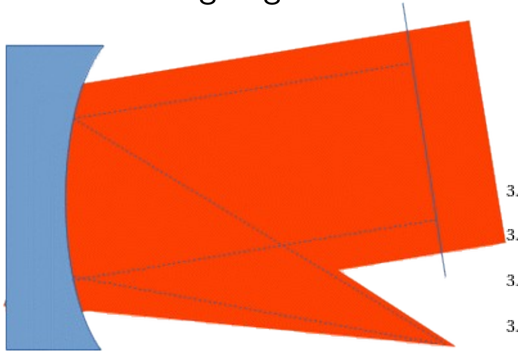


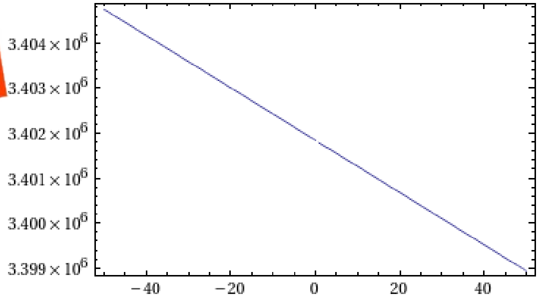
Focusing an ultrashort, high power laser pulse: Why using parabolic mirrors

Need for the usage of reflective focusing optics

Can we use a mirror of any shape?

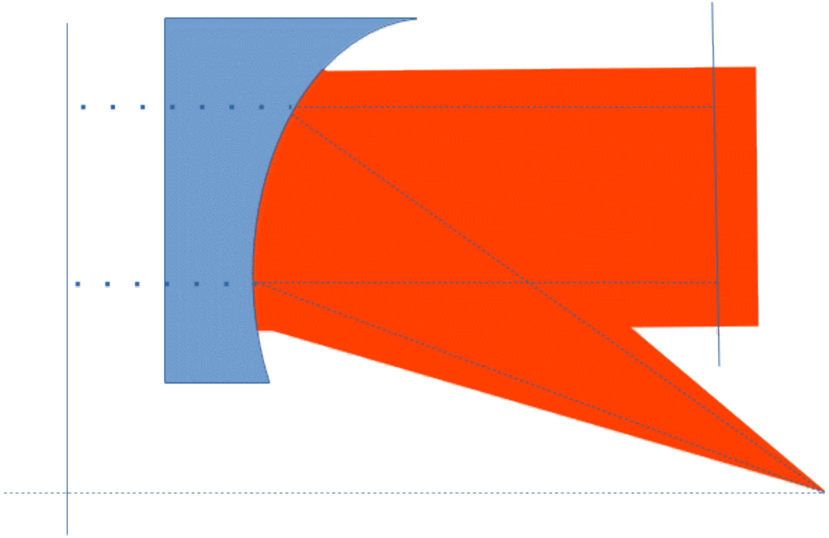
An example: spherical mirror with focal length 1m, beam with size 10cm, impinging on the mirror with a 1deg angle



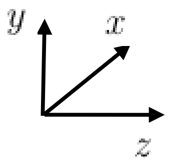


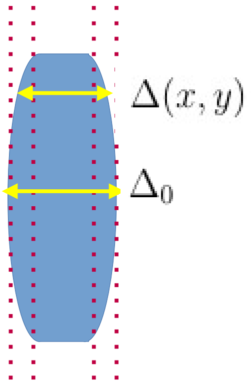
Pure geometrical optics, time of arrival (in fs) at the focal point of each ray

Parabolic mirrors are the only allowed surfaces to keep the original pulse duration!



Wavefront aberrations. Intro: A *thin* lens as a phase object





Thin lens: a ray exits with no transverse shift

Phase delay experienced by a small beamlet crossing the lens at (x,y)

$$\phi(x,y) = k\,n\,\Delta(x,y) + k_0\left[\Delta_0 - \Delta(x,y)\right]$$

The passage through the lens can thus be represented as a simple phase transformation, through the factor

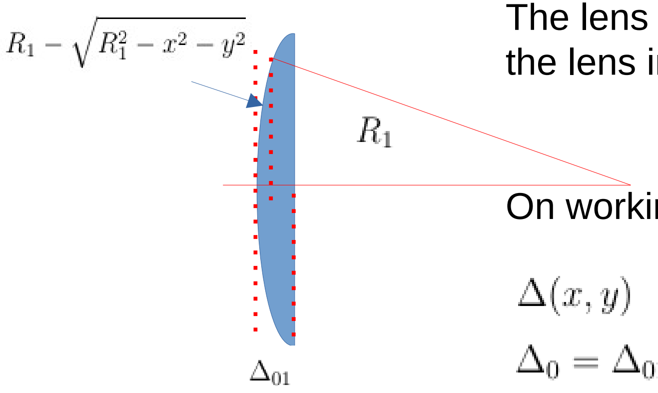
$$t_l\left(x,y\right) = \exp\left[ik\Delta_0\right]\exp\left[ik\left(n-1\right)\Delta\left(x,y\right)\right]$$

In other words, the (complex) field across a plane immediately behind (downstream of) the lens is related to that on a plane immediately before the lens by

$$U'\left(x,y\right) = t_e\left(x,y\right)U_l\left(x,y\right)$$

Thus, if a *plane* wave(front) is incident on the lens, the complex field just behind the lens can be written (neglecting constant phase terms) as

$$U'(x,y) = \exp\left[ik\left(n-1\right)\Delta\left(x,y\right)\right]$$



The lens thickness at a given point (x,y) can be calculated by decomposing the lens into 3 layers (convex-plane, plane-plane, plane-convex)

$$\Delta(x,y) = \Delta_1(x,y) + \Delta_2(x,y) + \Delta_3(x,y)$$

On working out the calculations, one gets

$$\Delta(x,y) = \Delta_0 - R_1\left(1 - \sqrt{1 - \frac{x^2+y^2}{R_1^2}}\right) - R_2\left(1 - \sqrt{1 - \frac{x^2+y^2}{R_2^2}}\right)$$

$$\Delta_0 = \Delta_{01} + \Delta_{02} + \Delta_{03}$$

Paraxial approximation $x^2 + y^2 \ll R^2$

$$\Delta(x,y) = \Delta_0 - \frac{x^2+y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Replacing this into the total phase shift (and defining the focal length in the well-known way), one arrives at

$$U'(x,y) = \exp\left[-i\frac{k}{2f}(x^2+y^2)\right]$$

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

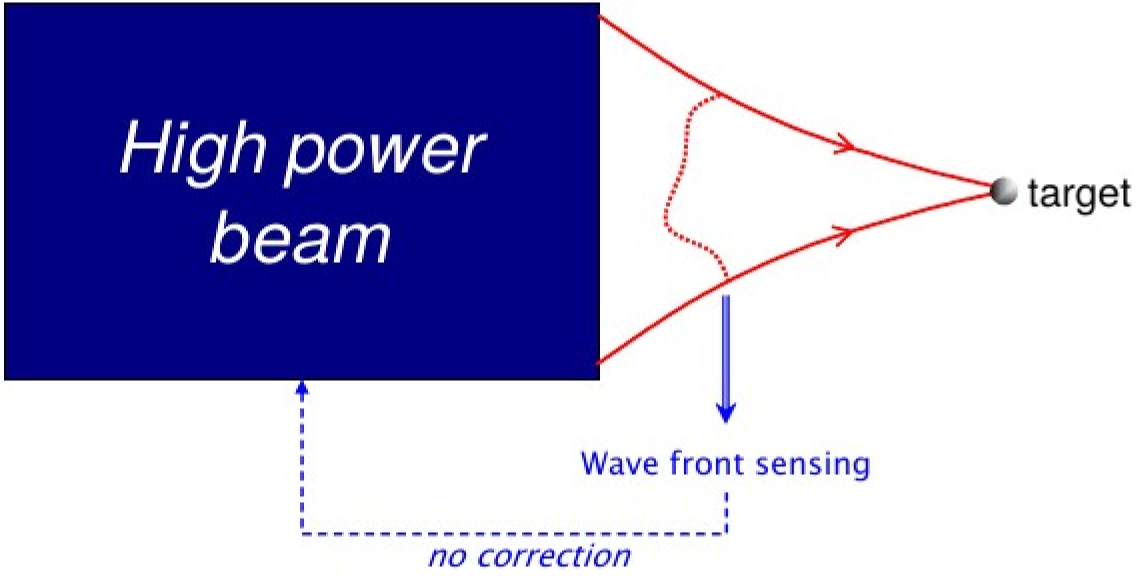
It can be easily demonstrated that this represents the wavefront of a spherical wave with “focus” at a distance *f.* The focusing can be thus seen as a pure phase transformation.

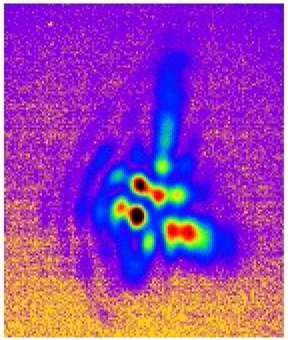
What does it happen when a non-planar wavefront is impinging on the lens?

Focusing an ultrashort, high energy beam down to ultrahigh intensity: Wavefront aberration issues

Ultrashort and ultraintense laser systems typically feature a large number of elements, whose optical performances can be greatly affected by material non-uniformity, manufacturing imperfections, material stresses, thermally induced deformations, …
All these factors can introduce wavefront aberrations, which in turn affect the energy distribution in the focal plane

Furthermore, aberrations introduced by the focusing device (most often an Off-Aaxis Parabolic mirror) can be measured and possibly corrected using active (deformable) mirrors





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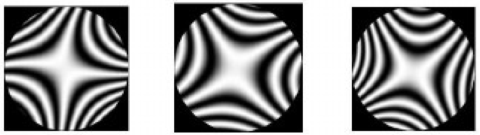
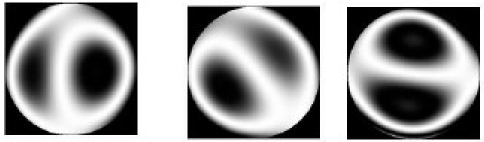
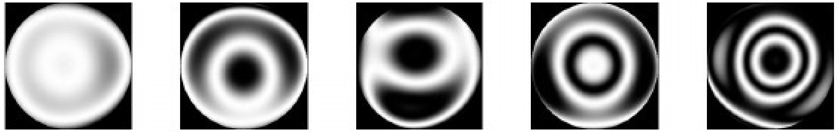
Wavefront (transverse phase) aberrations: Effects in the focal spot plane



Astigmatism: Magnitude [0/1] , angle 0, 45°, 90

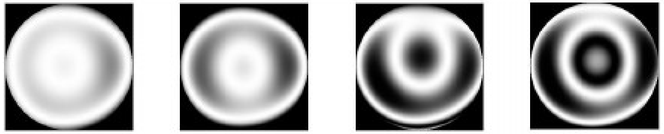


Coma: Magnitude [0/+1] 1, angle 0°, 45°, 90°

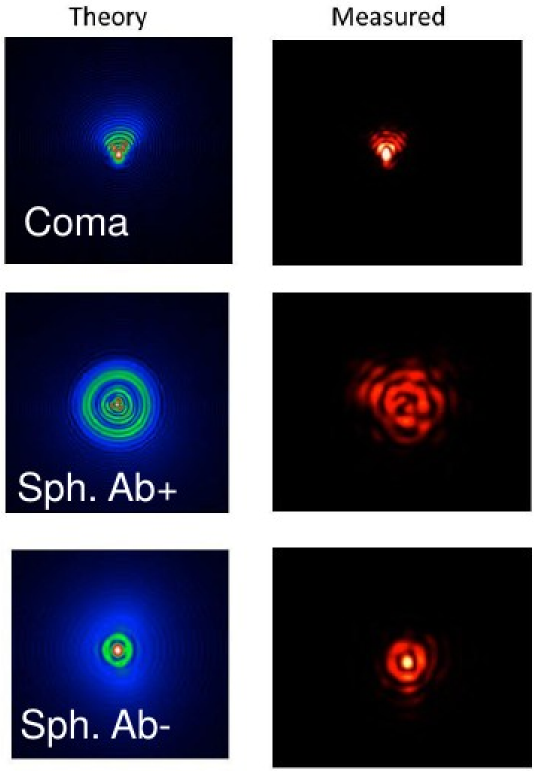


0.2 0.4 0.6 0.8 1

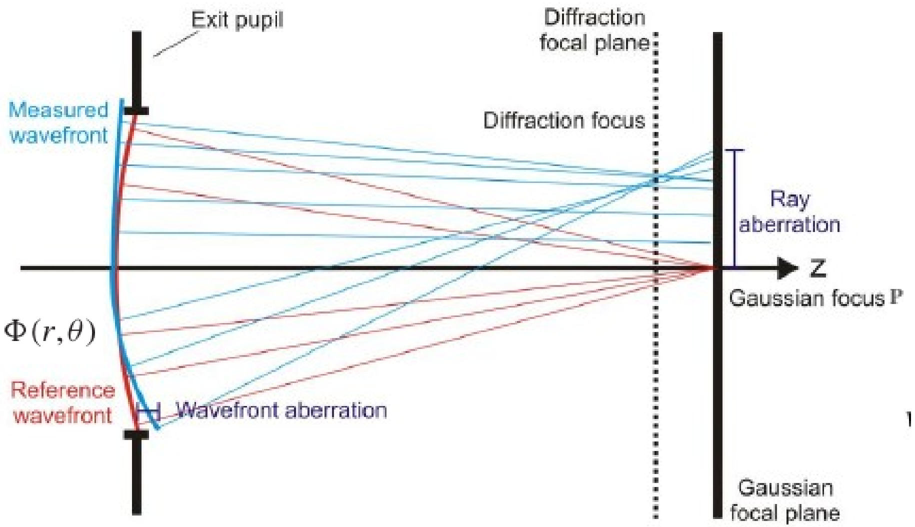
Spherical aberration [-1/+1]



-0.2 -0.4 -0.6 -0.8



Wavefront (transverse phase) aberrations: characterization and figure(s) of merit



Most of the time the deviation of the wavefront from a plane is described in terms of Zernike polynomials, which forms a complete set of polynomials onto the unit circle:

$$\Phi(r,\theta) = A_{00} + \frac{1}{\sqrt{2}}\sum_{n=2}^{\infty} A_{n0}R_n^0(\rho) + \sum_{n=1}^{\infty}\sum_{m=1}^{n} A_{nm}R_n^m(\rho)\cos(m\theta)$$

with

$$Z_n^m(\rho,\theta) = R_n^m(\rho)e^{im\theta}$$

$$R_n^m(\rho) = \frac{1}{2^k k!}\frac{1}{\rho^m}\left[\frac{1}{\rho}\frac{d}{d\rho}\right]^k\left[(\rho^2-1)^k\rho^{n+m}\right]$$

Strehl ratio: gives the maximum intensity achievable with an aberrated beam normalized to the one from an unaberrated one (see Born&Wolf for a deeper discussion)

$$i(P) = \frac{I(P)}{I^\star} = \frac{1}{\pi^2}\left|\int_0^1\int_0^{2\pi} e^{i[k\Phi(Y_1^\star,\rho,\theta) - v\rho\cos(\theta-\psi) - \frac{1}{2}u\rho^2]}\rho\, d\rho\, d\theta\right|^2.$$

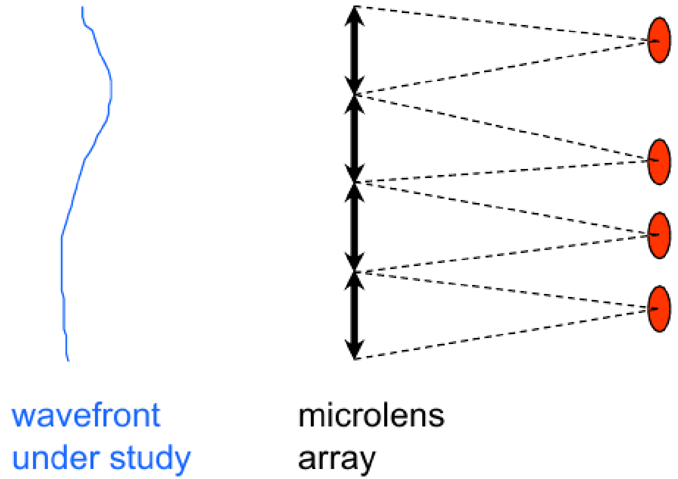
The Strehl ratio is related to the mean square deformation of the wavefront:

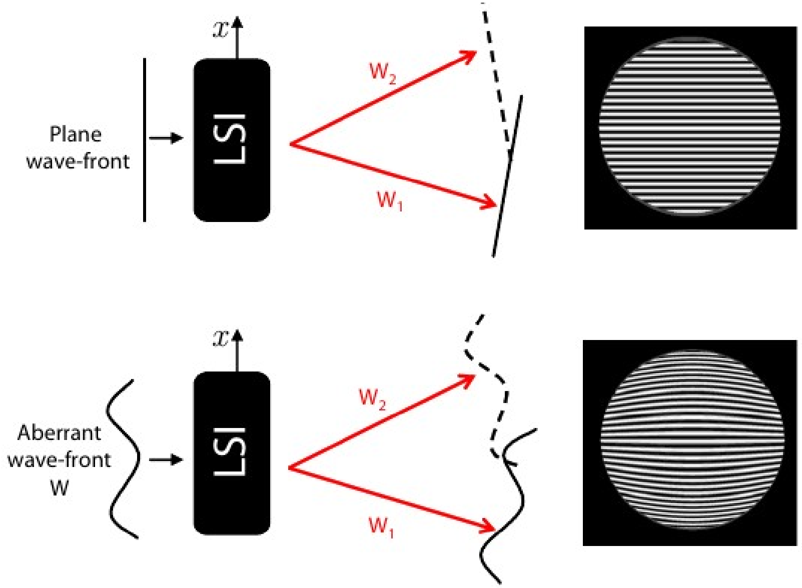
$$\frac{I}{I_0} = 1 - k^2\left[\overline{\Phi^2} - (\overline{\Phi})^2\right]$$

Wavefront (transverse phase) aberrations: Characterization techniques

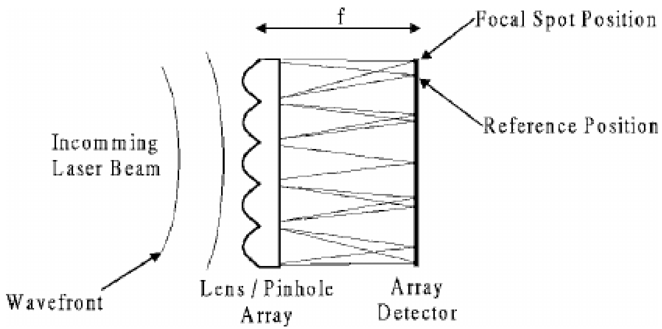
For the characterization of (ultra)short laser pulses, two types of WFS are most often employed

Shack-Hartmann WFS
Measures the displacements (with respect to an unaberrated beam) of the spot of different beamlets focused by an array of lenses





Wavefront (transverse phase) aberrations: The Shack-Hartmann WFS



The spot deviation is related to the slope of the deformed wavefront

The displacements of the spot centroids with respect to a plane wave reference position is a measure of the local Poyinting vector and thus the local gradient of the wavefront

$$\hat{\mathbf{S}}_\perp = \begin{pmatrix} \partial w/\partial x \\ \partial w/\partial y \end{pmatrix}_{ij} = \boldsymbol{\beta}_{ij} = \frac{1}{f}\begin{pmatrix} x_c - x_r \\ y_c - y_r \end{pmatrix}_{ij}$$

These data can be used to reconstruct the wavefront, starting from a polynomial expansion (for instance, a Zernike expansion)

$$\beta_{ij}^{(x,y)} = \frac{\partial w(x,y)}{\partial (x,y)_{ij}} = \sum_{k=1}^{L} c_k \frac{\partial P_k(x,y)}{\partial (x,y)_{ij}}$$

Data from SH WFS can also be used to calculate "standard" beam parameters, such as the M^2 parameter:

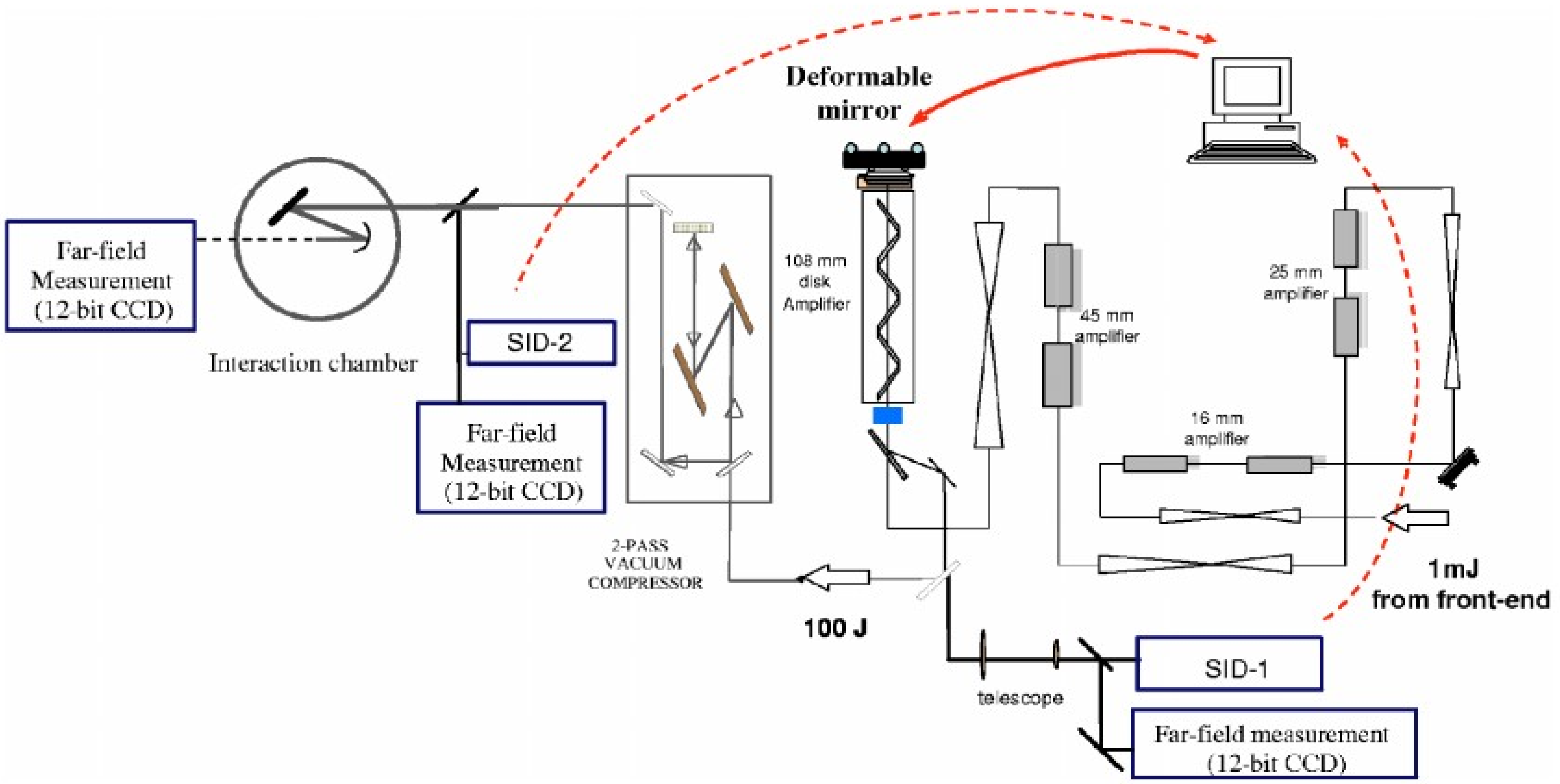
$$M_x^2 = 2k\sqrt{\langle x^2\rangle \cdot \langle u^2\rangle - \langle xu\rangle^2}$$

which depends on 2nd order moments of the so-called Wigner distribution:

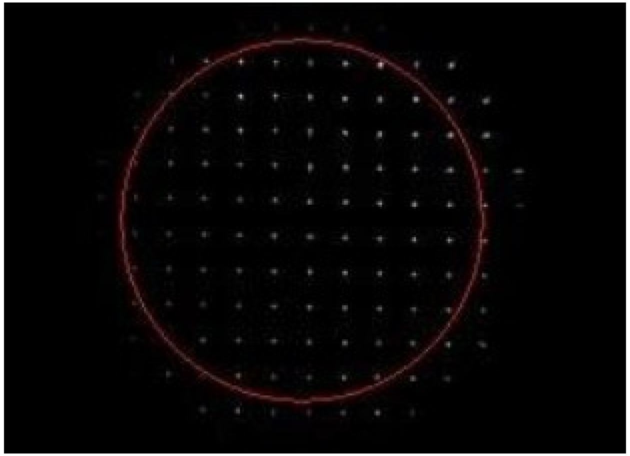
$$\langle x^2\rangle = \frac{\sum_{ij}(x_{ij} - \langle x\rangle)^2 I_{ij}}{\sum_{ij} I_{ij}} \qquad \langle xu\rangle = \frac{\sum_{ij}(\beta_{x,ij} - \langle\beta_x\rangle)\cdot(x_{ij} - \langle x\rangle)\cdot I_{ij}}{\sum_{ij} I_{ij}} \qquad \langle u^2\rangle = \frac{\sum_{ij}(\beta_{x,ij} - \langle\beta_x\rangle)^2 I_{ij}}{\sum_{ij} I_{ij}} + \frac{1}{k^2}\frac{\sum_{ij}(1/I_{ij})(\partial I/\partial x)_{ij}^2}{4\sum_{ij} I_{ij}}$$

For a complete discussion of the analysis of SH WFS see B. Schafer *et al*, Rev. Sci. Instrum. **77**, 053103 (2006)

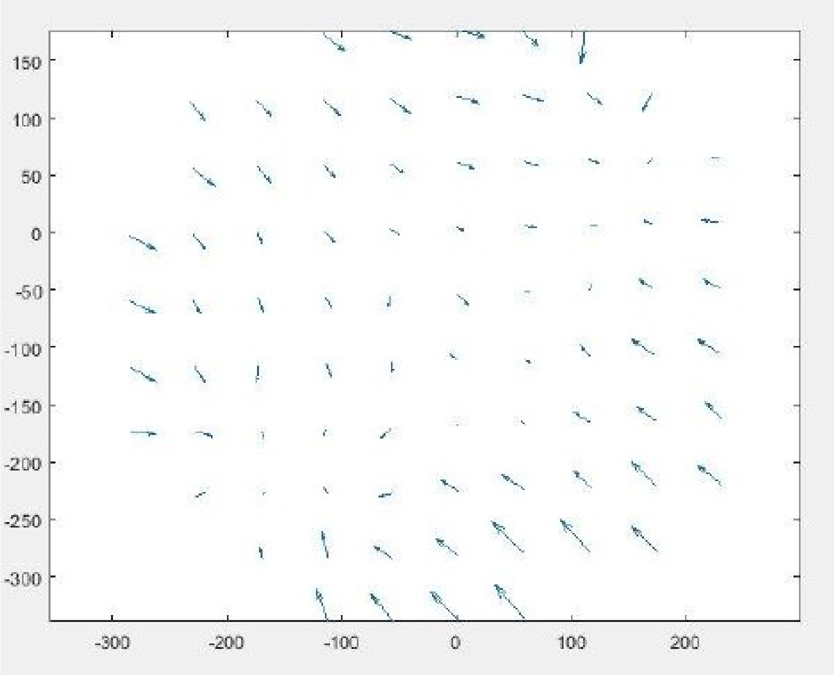
Correction of WF aberrations using deformable mirrors



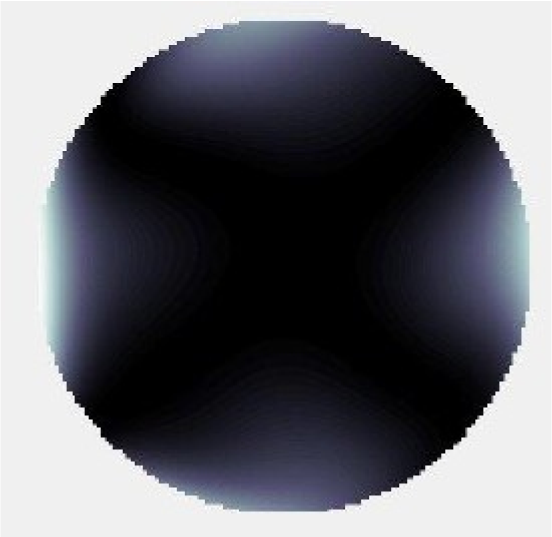
Raw image from SH detector

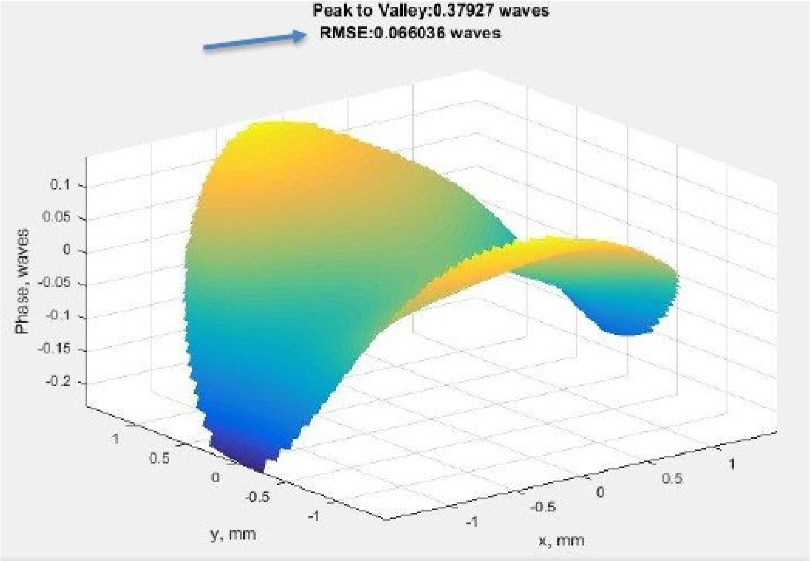


Retrieved local displacement from local reference centroid

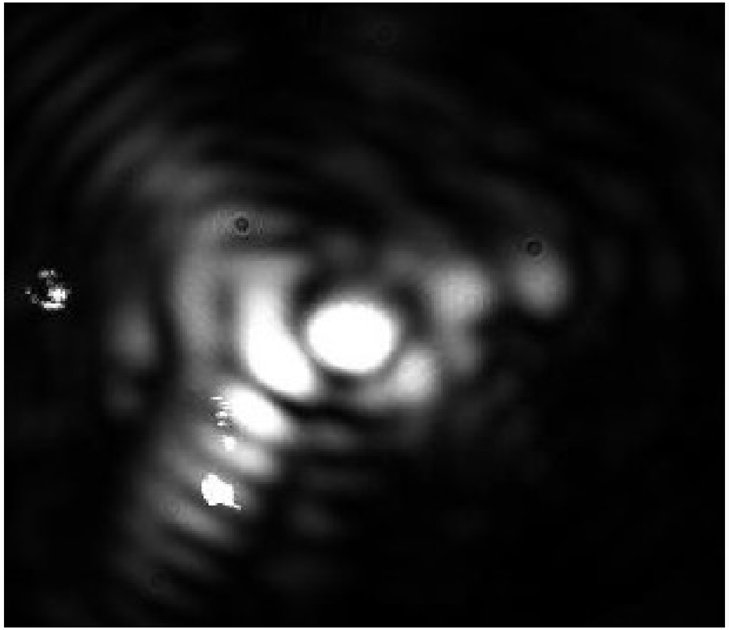


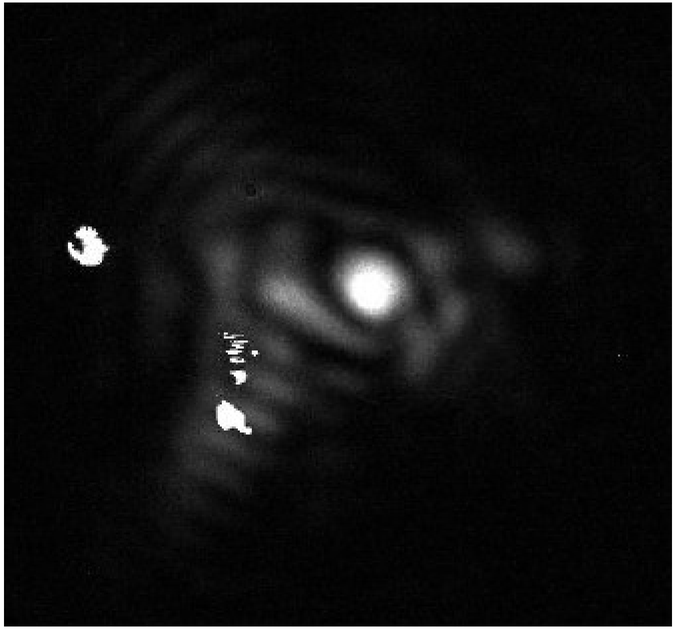
Retrieved wavefront





Vertical Astigmatism

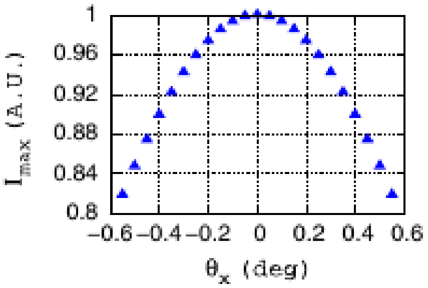


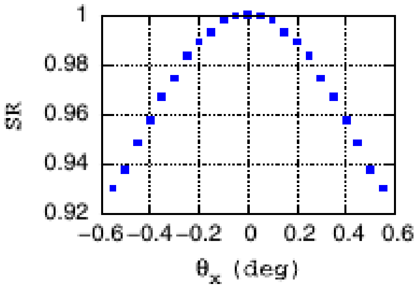


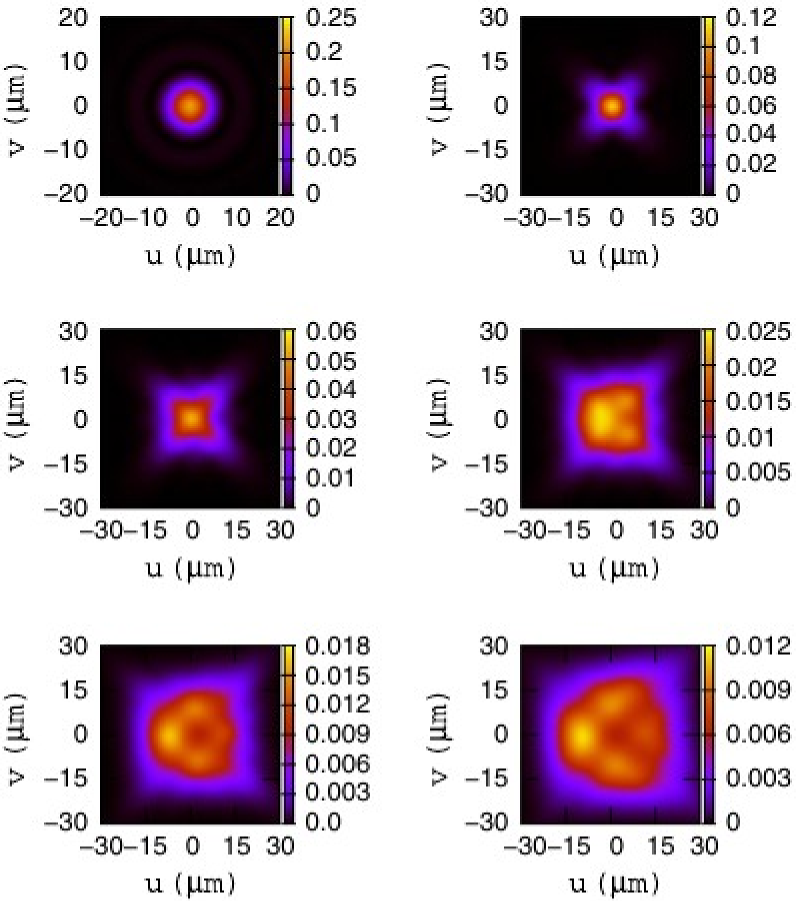
Wavefront aberrations introduced by incorrect OAP alignment

Wavefront distortions can also be introduced by incorrect Off-Axis Parabola (focusing optics) alignment

Even for very small angles of misalignment, this results in focal spot broadening and thus both maximum intensity and Strehl ratio degradation





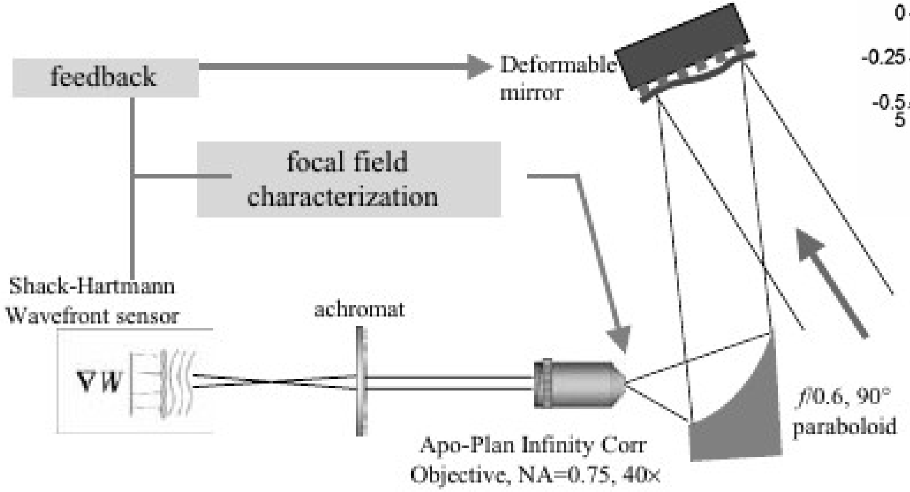


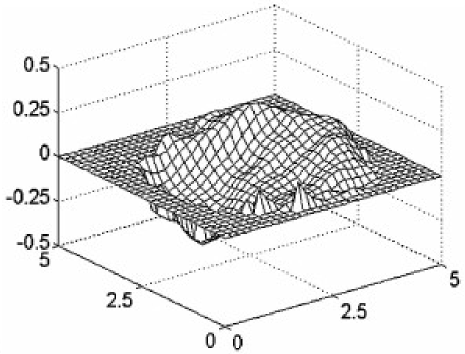
→ see Gabriele's talk

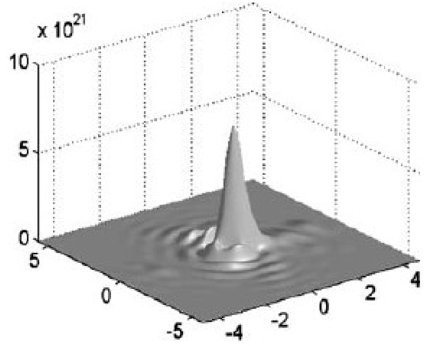
L. Labate *et al.*, Appl. Opt. **55**, 6506 (2016)

Wavefront aberrations introduced by incorrect OAP alignment (2)

Wavefront sensing and active correction with deformable mirrors can also be used to compensate for OAP misalignments







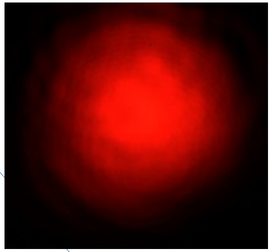
$I_{peak}=(6.9\pm0.7)\times10^{21}\ \mathrm{W/cm^2}$

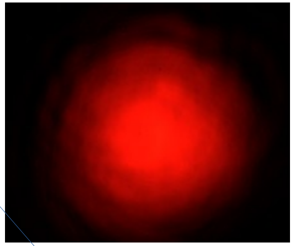
Characterization of focal field formed by a large numerical aperture paraboloidal mirror and generation of ultra-high intensity (10^{22} W/cm^2)

S.-W. BAHK[1,*]
P. ROUSSEAU[2]
T.A. PLANCHON[3]

Appl. Phys. B 80, 823–832 (2005)

Phase retrieval from intensity measurements





The above measurement techniques all required a “direct” characterization of the wavefront.
In 1972 Gerchberg and Saxton proposed a technique (actually already in use in other fields) to retrieve the phase from pure intensity measurements.

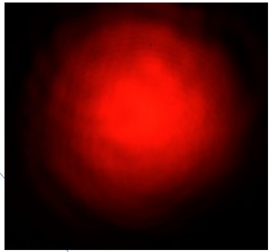
Let us consider 2 images of the beam, taken at 2 positions. By theoretical arguments, we know how the fields at the 2 positions are related to each other. For instance, in the paraxial approximation, I have a Kirchoff integral:

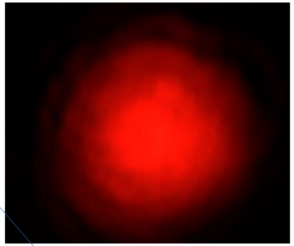
$$E(x,y,z) = \frac{k \exp\left(ik\,\frac{x^2+y^2}{2z}\right)}{2\pi i z} \iint_{-\infty}^{+\infty} E(x',y',0) \exp\left(ik\,\frac{x'^2+y'^2}{2z} - ik\,\frac{xx'+yy'}{z}\right) dx'\,dy'.$$

This equation provides me with a *propagation operator, P* (and its inverse *P-1*)

Phase retrieval from intensity measurements

I can thus retrieve the phase (wavefront) by pure intensity measurements using the following algorithm:





$$E^{(1)\prime} = \sqrt{I^{(1)}}e^{i\phi_0} \rightarrow E^{(2)} = P\left[E^{(1)\prime}\right] \rightarrow E^{(2)\prime} = \sqrt{I^{(2)}}\frac{E^{(2)}}{|E^{(2)}|}$$

$$E^{(2)\prime} = \sqrt{I^{(2)}}\frac{E^{(2)}}{|E^{(2)}|} \rightarrow E^{(1)} = P^{-1}\left[E^{(2)\prime}\right] \rightarrow E^{(1)\prime} = \sqrt{I^{(1)}}\frac{E^{(1)}}{|E^{(1)}|} \rightarrow E^{(2)} = P\left[E^{(1)\prime}\right]$$

The algorithm can either be stopped after a given number of iterations, or when a condition, encompassing a metric, is met. For instance, one can calculate the following quantity at each iteration

$$\hat{J} = \sum_i \iint_{-\infty}^{+\infty} \left(E^{(i)} - \sqrt{I^{(i)}}\right)^2 dx\, dy.$$